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Heat stress estimation as an inverse problem: variational approach.

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Abstract: Is shown a methodology for estimation of heat stress (changes in the spatial dependence of thermal conductivity) based on solving the associated inverse problem by a variational method (*Alifanov* approach) in the form of the adjoint equation. Three profiles of thermal conductivity's spatial dependence are considered: a step function (discontinuous), a continuous function with discontinuous derivative, and a sigmoid function (continuous function with continuous derivative).

keywords: Applications in Engineering and Nanoscience, Inverse Problems, Thermal Stress.

1. INTRODUCTION

Forward heat conduction problems are focused on determination of the temperature field on the material, when the boundary and initial conditions, heat source/sink terms (if any), and the physical properties of the material are known. On the other hand, an inverse heat conduction problem is concerned with the estimation of such quantities (boundary or initial conditions, source/sink terms, physical properties) from the temperature and/or heat flux measurements.

Mathematically, the Inverse problems are classified as ill-posed problems. The existence of a solution for an inverse heat transfer problem can be assured based on physical reasoning, but the requirement of uniqueness can only be formally proved for some special cases. Also, the inverse problem solution is generally unstable. Therefore, small perturbations in the input data, like random errors inherent to the measurements used in the analysis, can cause large oscillations on the solution.

The thermal conductivity on a material can present spatial dependency, in this case, the regime of heat conduction (based on conservation of heat and the Fourier law) can be given by:

Let $T(x,t)$, $k(x)$ and $f(x) \in L^2$ real functions of real variables, with:

$$\frac{\partial[\kappa(x)\frac{\partial T(x,t)}{\partial x}]}{\partial x} - \frac{\partial T(x,t)}{\partial t} = 0, x \in (0,1), t \in [0,\infty);$$

$$T(x,0) = f(x), x \in [0,1];$$

$$\kappa(x)\frac{\partial T(0,t)}{\partial x} = 0, t \in [0,\infty);$$
$$\kappa(x)\frac{\partial T(1,t)}{\partial x} = 0, t \in [0,\infty);$$

The spatial profile of the thermal conductivity may change due to heat stress suffered by the material. The objective of this study is: given a material and a set of measurements of temperature at any moment of its use, to estimate the thermal conductivity's spatial profile at this exact moment, and then compare it with the original profile – the difference between these profiles represents the effective thermal stress (thermal damage) suffered by the material.

In intent to solve this inverse problem will be applied the variational method - also known as conjugate gradient method (CGM) with the adjoint equation (Alifanov's approach) [1,2,3].

2. METHODOLOGY

The standard computational procedure of the CGM is briefly summarized in the following algorithm:

<u>Step 1:</u> To select an initial guess to the thermal conductivity's spatial profile.

<u>Step 2:</u> To use the direct problem as restriction's equation on possible solutions. Then is calculated the cost function: distance between the estimated solution and the experimental data.

<u>Step 3:</u> The problem is brought to the field of Lagrange multipliers ($\lambda = \lambda(x,t)$ - adjoint equation):

$$J[T(x,t)_{\kappa(x)}, \lambda(x,t)] = \sum_{m=1}^{m=M} \int_{t=0}^{t=\tau} [T(x_m,t) - T(x_m,t)^{\text{Exp}}]^2 dt - \int_{x=0}^{x=1}\int_{t=0}^{t=\tau} \lambda(x,t)[\frac{\partial[\kappa(x)\frac{\partial T(x,t)}{\partial x}]}{\partial x} - \frac{\partial T(x,t)}{\partial t}];$$

with the following sensitivity problem:

$$\frac{\partial[\Delta\kappa(x)\frac{\partial T(x,t)}{\partial x}]}{\partial x} + \frac{\partial[\kappa(x)\frac{\partial \Delta T(x,t)}{\partial x}]}{\partial x} - \frac{\partial \Delta T(x,t)}{\partial t} = 0, x \in (0,1), t \in [0,\infty);$$

$$\Delta T(x,0) = 0, x \in [0,1];$$

$$-\kappa(0)\frac{\partial T(0,t)}{\partial x} = \Delta\kappa(0)\frac{\partial T(0,t)}{\partial x}, t \in [0,\infty);$$

$$-\kappa(1)\frac{\partial T(1,t)}{\partial x} = \Delta\kappa(1)\frac{\partial T(1,t)}{\partial x}, t \in [0,\infty);$$

Since,

$$\Delta J(x,t) = \int_{x=0}^{x=1}\int_{t=0}^{t=\tau} J'(x,t)\Delta\kappa(x)dxdt;$$

the gradient of the cost function, can be found by:

$$J'(x,t) = -\frac{\partial\lambda(x,t)}{\partial x}\frac{\partial T(x,t)}{\partial x};$$

where,

$$\sum_{m=2}^{m=M-1}\int_{t=0}^{t=\tau} 2[T(x_m,t) - T(x_m,t)^{\text{Exp}}]^2 dt\delta(x - x_m) + \frac{\partial[\kappa(x)\frac{\partial\lambda}{\partial x}]]}{\partial x} + \frac{\partial\lambda}{\partial t} = 0;$$

$$\lambda(x,\tau) = 0, x \in (0,1);$$

$$-\frac{\partial\lambda(0,t)}{\partial x} = 2[T(1,t) - T(1,t)], t \in [0,\infty);$$

$$\frac{\partial\lambda(1,t)}{\partial x} = 2[T(0,t) - T(0,t)], t \in [0,\infty);$$

Step 4: Determination of the proper terms of the GCM: direction of descent (p^k) and conjugate coefficiente (γ):

$$p^k = -\nabla J^k \quad \text{se} \quad k = 0$$

$$p^k = -\nabla J^k + \gamma p^{k-1} \quad \text{se} \quad k = 1, 2, \ldots$$

with,

$$\gamma = \frac{\|\nabla J^k\|_2^2}{\|\nabla J^{k-1}\|_2^2}.$$

Step 5: Inline search in the direction of p^k, finding the step size β:

$$\beta = Arg \min \varphi(r),$$

$$\varphi(r) = J(\boldsymbol{\mathcal{K}}^k + r\,\boldsymbol{p}^k).$$

Step 6: Update the estimated solution:

$$\boldsymbol{\mathcal{K}}^{k+1} = \boldsymbol{\mathcal{K}}^k + \beta\,\boldsymbol{p}^k.$$

Step 7: Increment the variable count of iterations, k, and to evaluate the stopping criterion (type Morosov [3]). In all simulations was used 5% of multiplicative gaussian noise in synthetic data.

3. EXPECTED RESULTS

Three profiles of thermal conductivity's spatial dependence are considered:

i) a step function (discontinuous function)
ii) a continuous function with discontinuous derivative
iii) a sigmoid function (continuous function with continuous derivative).

The first profile corresponds to the simulated project configuration, the second profile to the first simulated damaged configuration and the second simulated damaged profile is the third profile. The estimation of these three profiles is shown, as like the especial features of each process. Will be also analyzed the robustness of the method against the number of experimental measurements, and level of noise, and the possibility of attacking the problem by a hierarchical approach [4].

References

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