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ESTIMATING CHAOS CONTROL PARAMETERS FROM TIME SERIES

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Abstract: We present a simple method to analyze time series, and estimate the parameters needed to control chaos in dynamical systems. Application of the method to a system described by the logistic map is also shown. Analyzing only two 100-point time series, we achieved results within 2% of the analytical ones. With these estimates, we show that OGY control method successfully stabilized a period-1 unstable periodic orbit embedded in the chaotic attractor.

keywords: Control of Chaos and applications, Time series analysis.

1. INTRODUCTION

Chaotic behavior is a common feature found in a wide range of complex systems of interest to science and engineering. The time evolution of a chaotic system is unpredictable yet deterministic and not random. Chaotic oscillations in a dynamical system may be reduced or even suppressed by chaos control techniques that disturb the system slightly. In this paper, we present a simple way to estimate the control parameters of OGY chaos control method through an analysis of the chaotic time series output by the system.

Chaos control techniques allows us to keep a chaotic system in a unstable periodic orbit embedded within a strange attractor with the application of small perturbations to the underlying dynamics. Due to the sensitive dependence on the actual state of the system, a small perturbation may cause dramatic changes in the system evolution. Owing precisely to the chaotic behavior of the system, its control turns out to be surprisingly efficient because every small region in the phase space of a strange attractor is crossed by orbits that visit every other regions in the attractor.

In this context, the OGY control method is a very simple one, relying on a variation of linear control with feedback proportional to the system output[1]. In a codimension-1 map

$$\mathbf{x}_{n+1} = \mathbf{F}\left(\mathbf{x}_n, r_n\right), \tag{1}$$

the controls needed to stabilize a period-1 unstable periodic orbit (fixed point) are small disturbances Δr_n around the nominal value r_0 of the control parameter r:

$$\Delta r_n = r_n - r_0 = -\gamma \hat{\mathbf{n}} \cdot \left(\mathbf{x}_n - \mathbf{x}_0^*\right) \tag{2}$$

where $\hat{\mathbf{n}}$ is a unit vector normal to the Poincaré section Σ in the neighborhood of the unstable fixed point $\mathbf{x}_0^*$, and γ is a proportional gain.

Applied to the stabilization of a fixed point in the logistic map[2]

$$x_{n+1} = F(x_n, r) = r x_n \left(1 - x_n\right), \tag{3}$$

OGY control method is based on the linearized map around the unstable fixed point $x^* = 1 - 1/r_0$, and around the nominal control parameter r_0,

$$x_{n+1} \approx x^* + \alpha \left(x_n - x^*\right) + \beta \left(r_n - r_0\right) \tag{4}$$

that leads to a modified map

$$x_{n+1} = \left(r_0 - \gamma \left(x_n - x^*\right)\right) x_n \left(1 - x_n\right) \tag{5}$$

when the system is close enough of the unstable fixed point x^*. Optimal control is achieved by choosing $\gamma = -\alpha/\beta$. Therefore, one must determine experimentally the location of the fixed point x^*, and the sensitivities

$$\alpha = \left.\frac{\partial F}{\partial x}\right|_{\substack{x=x^* \\ r=r_0}} \tag{6}$$

$$\beta = \left.\frac{\partial F}{\partial r}\right|_{\substack{x=x^* \\ r=r_0}} \tag{7}$$

in order to apply controls on the system.

Reconstruction of some aspects of the chaotic dynamics from time series is a subject explored through a variety of methods[3–5]. We introduce a very simple recurrence method to estimate the parameters required in order to apply the OGY method to control chaos in dynamical systems. For ease of presentation, we restrict this discussion to unidimensional maps as the logistic map, and to period one fixed points. Therefore, it suffices to estimate the values of x^*, α, and β from a time series.

1.1. Our Proposal

A linearized version of the map around $x = x^*$, and $r = r_0$ may be written as

$$x_{n+1} \approx x^* + \alpha \left(x_n - x^*\right) + \beta \left(r_n - r_0\right). \tag{8}$$

Next, we fix the value of the control parameter to the nominal value r_0, and select only those situations in which the variation

$$\Delta_n = x_{n+1} - x_n \approx (\alpha - 1)(x_n - x^*) \qquad (9)$$

is small (less than some ε, say). In these cases, the system is in a small neighborhood of the fixed point due to the slow dynamics of the map around x^*. Then, it is straightforward to show that

$$x_0^* \approx \left\langle \frac{x_n \Delta_m - x_m \Delta_n}{\Delta_m - \Delta_n} \right\rangle \qquad (10)$$

where Δ_m is the corresponding variation for another close encounter in the neighborhood of the fixed point, and the brackets indicate averaging over many such close encounters provided by a long time series. Despite the simplicity of this recurrence method, fixed point detection may be enhanced by techniques that manipulates the probability distribution on the attractor through fixed point transforms[4].

Having an estimate for x^*, it is easy to show that

$$\alpha \approx 1 + \left\langle \frac{\Delta_n}{x_n - x^*} \right\rangle. \qquad (11)$$

Finally, in order to estimate β, we vary the control parameter to a value r_1 slightly different from r_0, and estimate the new fixed point x_1^* using Eq. (10). Hence, averaging over many different values of r_1, we discover that

$$\beta \approx (1-\alpha) \left\langle \frac{\delta x^*}{\delta r} \right\rangle = (1-\alpha) \left\langle \frac{x_1^* - x_0^*}{r_1 - r_0} \right\rangle. \qquad (12)$$

2. RESULTS AND DISCUSSION

To demonstrate the power of this simple method, we estimate x^*, α, β, and γ for the logistic map with only two runs of 100 points each. In these simulations, we set $r_0 = 3.90$ and $r_1 = 3.91$, right in the middle of a heavily chaotic region, and we also fixed the radius for a close encounter to be detected in 10% of the total variation of the logistic map. The results are shown in Table 1. Even these short few runs were enough for a fair estimation of the parameters needed to control the chaos in the system.

Table 1 – Theoretical and simulated values for the fixed point x^*, sensitivities α and β, and proportional gain γ.

Parameter	x^*	α	β	γ
Theory	0.74359	-1.90	0.1907	9.97
Simulation	0.74347	-1.86	0.1892	9.85
Error	0.016%	2.1%	0.79%	1.2%

Using the estimated parameters, we were able to stabilize a period-1 unstable periodic orbit embedded in the heavily chaotic region of the logistic map as shown in Fig. 1. In order to keep the orbit stabilized, additional controls amounting to only $\Delta r / r_0 = 0.074\%$ had to be continuously applied.

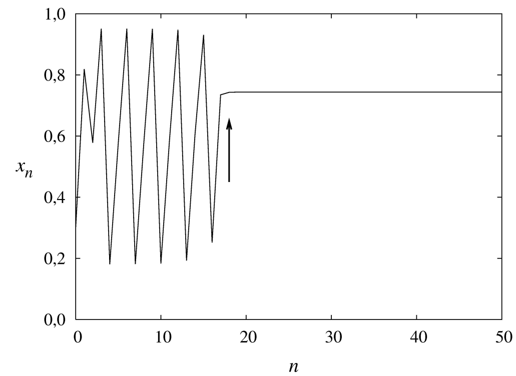

Figure 1 – Stabilized fixed point in the logistic map ($r_0 = 3.90$) using OGY chaos control method with parameters estimated from time series. The arrow marks the first time that controls were applied.

Time series are tipically used as a means to gain insight on the general dynamics of a chaotic system, including Lyapunov exponents, and some topological characteristics. However, from a control engineering perspective, this kind of information is excessive. The novelty of the proposed method is to analyze the time series to extract just the information required to control chaos, drawing attention only to the dynamics on a small neighborhood of the unstable periodic orbits.

3. CONCLUSION

We presented a simple way to estimate the set of parameters needed in order to control chaos. The method applied here to stabilize a fixed point in the logistic map may be easily adapted to stabilize a period-k unstable periodic orbit in higher dimensional systems where α is replaced by the jacobian matrix $[\alpha_{ij}] = [\partial F_i / \partial x_j]$, and β is replaced by a matrix of sensitivities $[\beta_{ij}] = [\partial F_i / \partial r_j]$. Analysis of only two very short time series of the logistic map was enough to estimate the control parameters within 2% of its analytical values. With these estimates, OGY method could successfully stabilize an unstable periodic orbit.

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