A Simple Method to Calculate the Maximum Usable Frequency
Jonas R. Souza, Inez S. Batista, Renata G. D. F. Costa, Divisão de Aeronômia/INPE, Brazil

Method to calculate the MUF
The ionospheric MUF can be calculated by

\[ MUF = M(D)foF2, \] (1)

where \( M(D) \) is the M-factor for a hop equal to \( D \).

A simple way to determine M-factor is using a spherical geometry model. The M-factor obtained with such geometry is given by

\[ M(D) = \sec \left( \arcsin \left( \frac{R_e \cos \theta}{R_e + h' mF2} \right) \right), \] (2)

where \( R_e \) is the Earth’s radius and \( \theta \) is the elevation angle, as can be seen in Figure 1. This Figure illustrates the geometry used to calculate M-factor.

The procedure presented above is not accurate due to the ionosphere refraction. To improve the M-factor calculation, we have considered the ionospheric effects which cause a time delay in the signal propagation and, consequently, an error in M-factor values obtained by only considering spherical geometry. This means that M-factor must be calculated for a virtual \( h' mF2 \) value which can be given by

\[ h' mF2 = h mF2 + \Delta h, \] (3)

where \( \Delta h = \frac{40.3}{(foF2)^2} \) TEC and TEC' is the total electron content below \( h mF2 \). Considering such ionospheric effect the new M-factor can be expressed as

\[ M'(D) = \sec \left( \arcsin \left( \frac{R_e \cos \theta}{R_e + h' mF2} \right) \right). \] (4)

Here, \( \theta \) is the elevation angle for a signal reflected at \( h' mF2 \). This methodology works for \( D = 3000 \) km. The M-factors, for hops lesser than 3000 km, were calculated by the algorithm published by Lookwood, 1983. The Lookwood’s algorithm uses M(3000), foF2, and E-layer critical frequency (foE) as input parameters.

Data and method validation
We compare our MUF results with observations collected over two equatorial stations São Luís (2.3°S; 44°W) and Fortaleza (4°S; 38°W) and also over a low-latitude location Cachoeira Paulista (22.5°S; 45°W) both on the Brazilian territory. The data were registered by digisondes, except for Fortaleza that was used an ionosonde. All data are representative of equinocial conditions and for geomagnetically quiet periods.
The MUF calculated for hops lesser than 3000 km were also validated. Figures 3 and 4 show MUFs for solar minimum and solar maximum respectively. Top panels are the results for hop = 200 km, middle panels are for hop = 600 km and the bottom panels present the results for hop = 1500 km. The MUFs calculated applying only the spherical geometry model show significant differences when compared with the ionogram data, except for hop = 200 km in which the differences basically disappear. The results are overestimated at the most of the times during both solar minimum and solar maximum periods. Since the M-factor tends to be equal 1 for small hops, the nice agreement for the lowest hop value was already expected. All MUF calculations, using the combination of our methodology with the Loockwood’s method, present excellent agreement with the data. Such validation was analyzed for different solar flux levels, different hops and locations, as we can see in Figures 3 and 4.

**MUF maps over Brazil**

Since we have a well established method to determine MUFs, we decide to examine the performance of a Parameterized Regional Ionospheric Model, PARIM. This model was developed to calculate 3D electron density over Brazil and part of South America (Souza et al., 2010, 2013). Here, PARIM was run to obtain the values of foE, foF2 and hmF2 which are the standard inputs of our method.

Figure 5 and 6 show MUF maps constructed with inputs given by PARIM for two solar flux levels, solar minimum and solar maximum, respectively. The results are also representative of equatorial conditions. The three columns present MUFs for hop of 600, 1500 and 3000 km, respectively. At each column there are results for 09, 15 and 21 Universal Time (UT). The magenta line crossing the maps is the position of the magnetic equator. The presence of the equatorial ionization anomaly is very clear for both solar minimum and solar maximum conditions, mainly during evening time.
Figure 3: Diurnal variations of the MUFs calculated with and without corrections of the ionospheric effects and experimental data for solar minimum and for hops of 200 (first line), 600 (second line) and 1500 km (third line).

Figure 4: Same as Figure 3, but for solar maximum conditions.

One interesting point to be noticed occurs when we have strong equatorial ionization anomaly development, as for example during high solar activity as presented in Figure 6, it is not possible to establish communication in east-west direction with a frequency range near those from anomaly crests. Obviously, for locations below the anomaly crests the east-west communications are possible using such frequencies.

Unfortunately, we did not have enough experimental data available to do MUF maps to compare them with our results. A full validation, i.e., validation covering all Brazilian places is necessary and it will be presented in a future publication. The uncertainties on the maps calculated using PARIM are in the uncertainties in the parameters foE, foF2 and hmF2 given by the PARIM itself. On the other hand, the values of foE, foF2 and hmF2 calculated for Conjugate Point Experiments
(COPEX campaign) presented good agreement with data, as published by Souza et al. 2013.

Conclusions
The main conclusion of this work may be summarized as follows:

(i) We have described a method to calculate MUF using foE, foF2 and hmF2. The method can be easily coupled to physical, empirical or parameterized models to make MUF predictions.

(ii) The MUFs calculated by applying only spherical geometry showed overestimate
values when compared with experimental data.

(iii) MUF calculated using spherical geometry and including the ionospheric effect corrections presents good agreement with data for a hop of 3000 km.

(iv) PARIM model has presented coherent predictions of MUFs for the Brazilian sector during both solar minimum and solar maximum conditions.

Acknowledgments

We would like to acknowledge support received from CNPq as fellowship of the Programa de Capacitação Institucional given to Renata G. D. F. Costa.

References


