



SEARCH OPTIMAL TRAJECTORIES USING EVGA AND SOLAR ELECTRIC PROPULSION

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Abstract: The spacecraft propulsion system have passed for diverse evolutions, leaving combustion engines and arriving at ion propulsion. The necessity of more efficient rockets stimulated the research in this scope. In this work ΔV will be analyzed proceeding from an electric propellant acting in set with gravitational maneuvers. The optimization of maneuvers will be approached in interplanetary missions using solar electric propulsion and Gravity Assisted Maneuver attended to reduce the costs of the mission. The high specific impulse of electric propulsion makes a Gravity Assisted Maneuver 1 year after departure convenient. Missions for several Near Earth Asteroids will be considered. The analysis suggests criteria for the definition of initial solutions demanded for the process of optimization of trajectories.

Trajectories for the asteroid 19989UQ are analyzed. Direct trajectories, trajectories with 1 gravity assisted from the Earth and with 2 gravity assisted from the Earth and either Venus are present. An indirect optimization method will be used in the simulations.

Keywords: Astrodynamics, Celestial Mechanics, Space Trajectories, Flyby, Solar Electric Propulsion.

1. INTRODUCTION

The solar electric propulsion could be the best option for the transports of the future due to its high specific impulse when compared to the chemical propulsion. Electric propellants are being extensively used to assist the propulsion of terrestrial satellites for the maneuvers of orbit correction and as primary propulsion in missions toward other bodies of the solar system.

Both NASA and ESA have launched spacecrafts which used SEP (Solar Electric Propulsion) as the primary propulsion system; NASA's DS1 and ESA's Smart-1 to the moon to comet Borrelly.

Indirect optimization methods are suitable for the low thrust trajectories that are used in simulations. A finite force is applied during a finite interval of time and it is necessary to integrate the state equation along the time to know its

effect. Several results exist in literature, starting with the works of Tsien (1953) and Lawden (1955). Other results and references can be found in Prado (1989), Prado and Rios-Neto (1993), Casalino and Colasurdo [1], Santos [2]. The most used method in this model is the so called "primer-vector theory", developed by Lawden (1953 and 1954) [4], [5], according to Prado [6], [7], Santos [8], [9]. In this paper, theory of optimal control is applied and a procedure based on the Newton Method to decide the boundary problems is developed. The Pontryagin's Maximum Principle (PMP) is used to maximize the Hamiltonian associated to the problem and evaluates the optimal structure of the "switching function".

The spacecraft leaves the Earth's sphere of influence with a hyperbolic velocity whose optimal magnitude and the direction will be supplied by the optimization procedure. The initial mass is directly related to the magnitude of the hyperbolic velocity, assuming that a chemical thruster is used to leave a low Earth orbit (LEO). Out of the Earth's sphere of influence, the electric propellants is activate and the available power is proportional to the square of the distance from the sun; the propulsion is provided by one or two "PPS 1350 ion thrusters and Phall1 thrusters (UNB)".

2. DESCRIPTION OF THE PROBLEM

The spacecraft will be considered a point with variable mass m and trajectory will be analyzed using the patched-conics approach. The time required by the spacecraft to leave the Earth's sphere of influence is neglected and, in this formulation, only equations of motion in the heliocentric reference system will be considered. The spacecraft is influenced by the Sun gravitational acceleration $\vec{g}(r)$ and the propulsion system of the vehicle implements a thrust T . With this formulation, a maneuver of Earth flyby can be used to gain energy and velocity, that provokes a discontinuity in the relative state variables in the velocity.

The variables are normalized using the radius of the Earth's orbit, the corresponding circular velocity, and the

mass of the spacecraft in stationary orbit as values of reference. The solar electric Propulsion will be considered, therefore, the available power and thrust varies with the square of the distance from the sun.

In the problem, the thrust is the only control during the heliocentric arcs, and it will be optimized to get the minimum consumption, that is measured by the final mass of the spacecraft. Since the thrust appears linearly in the equation of motion, a bang-bang control, that consists of alternating ballistic arcs with arcs of maximum thrust will be required. The trajectory is composed by a succession of ballistic arcs (zero-thrust) and arcs of maximum thrust, where the optimal direction will be supplied by the optimization procedure.

The boundary conditions are imposed in satisfactory way at the junctions between trajectory arcs.

The integration initiates when the spacecraft leaves the Earth's sphere of influence, at the position $\vec{r}_i = \vec{r}_\oplus(t_i)$ that coincides with the Earth is position, considering the velocity \vec{v}_i free. The hyperbolic velocity is given by $\vec{v}_{\infty i} = \vec{v}_i - \vec{v}_\oplus(t_i)$, assuming that a rocket thruster is used to leave the Low Earth Orbit (LEO) with an impulsive maneuver; the vehicle mass on LEO is specified. The increment of velocity (ΔV) demanded to provide the hyperbolic velocity is $\Delta V = \sqrt{v_{\infty i}^2 + v_e^2} - v_c$, where v_e and v_c are the escape and circular velocity at the LEO radius [1].

The initial mass at the exit from the Earth's sphere of influence is,

$$m_i = a - bV_\infty - cV_\infty^2 \quad (1)$$

where,

$\varepsilon(1 - m_i)$ is the jettisoned mass of the exhausted motor, which is proportional to the propellant mass. The spacecraft intercepts the Earth and accomplishes Gravity Assisted Maneuvers [6], [9]. The position of the vehicle $\vec{r}_\pm = \vec{r}_\oplus(t_\pm)$ is constrained and the magnitude of the hyperbolic excess velocity $\vec{v}_{\infty\pm} = \vec{v}_\pm - \vec{v}_\oplus(t_\pm)$ is continuous $v_{\infty+}^2 = v_{\infty-}^2$ [1].

If the minimum height constraint on the flyby is requested, a condition on the velocity turn angle is added:

$$\vec{v}_{\infty+}^T \vec{v}_{\infty-} = -\cos(2\phi) v_{\infty-}^2 \quad (2)$$

where,

$$\cos(\phi) = \frac{v_p^2}{(v_{\infty-}^2 + v_p^2)} \quad (3)$$

v_p is the circular velocity at the low distances allowed for a planet.

$$\vec{v}_{\infty\pm} = \vec{v}_{i\pm} - \vec{v}_4 \quad (4)$$

At the final point (subscript f), the position and velocity vectors of the spacecraft and the asteroid coincide,

$$\mathbf{r}_f = \mathbf{r}_A(t_f) \quad (5)$$

$$\mathbf{v}_f = \mathbf{v}_A(t_f) \quad (6)$$

The theory of optimal control provides the control law and necessary boundary conditions for optimality.

3. OPTIMIZATION PROCEDURES

The objective is to use the theory of optimal control to maximize the spacecraft final mass.

Dynamical equations are,

$$\begin{aligned} \dot{\vec{r}} &= \vec{v} \\ \dot{\vec{v}} &= \vec{g}(\vec{r}) + \frac{\vec{T}}{m} \\ \dot{m} &= -\frac{\vec{T}}{c} \end{aligned} \quad (7)$$

Applying the theory of optimal control, the Hamiltonian function is defined as (Lawden, 1954) [3, 4]:

$$H = \vec{\lambda}_r^T \vec{v} + \vec{\lambda}_v^T (\vec{g} + \frac{\vec{T}}{m}) - \lambda_m \frac{\vec{T}}{c} \quad (8)$$

An indirect optimization procedure is used to maximize the payload. According to Pontryagin's Maximum Principle the optimal controls maximize H.

The nominal thrust T_0 at 1 AU, and the electrical power are [1],

$$\begin{aligned} P_0 &= \frac{T_0 c}{2\eta} \\ T_{Max} &= \frac{T_0}{r^2} \end{aligned} \quad (9)$$

Optimal control theory provides differential equation for the adjoint equations of the problem (Euler-Lagrange).

The Adjoint equations are,

$$\dot{\vec{\lambda}}_r^t = \vec{\lambda}_v^t \frac{\partial \vec{g}}{\partial \vec{r}} - S_f \frac{\partial T}{\partial \vec{r}} \quad (10)$$

$$\dot{\vec{\lambda}}_v^t = -\vec{\lambda}_r^t \quad (11)$$

$$\dot{\vec{\lambda}}_r^t = \vec{\lambda}_v^t \frac{\vec{T}}{m^2} \quad (12)$$

where, $G = \frac{\partial \vec{g}}{\partial \vec{r}}$.



Optimal control: thrust direction and magnitude are,

$$\begin{aligned} \vec{T} // \vec{\lambda}_v \\ H = \vec{\lambda}_r^t \vec{v} + \vec{\lambda}_v^t G + \vec{T} \left(\frac{\lambda_v}{m} - \frac{\lambda_m}{c'} \right) \\ S_f = \frac{\lambda_v}{m} - \frac{\lambda_m}{c'} \end{aligned} \quad (13)$$

where,

c' - is the effective exhaust velocity of the rocket thruster;

$$T_{Max} = \begin{cases} T_0 \rightarrow S_f > 0 \\ 0 \rightarrow S_f < 0 \end{cases} \quad (14)$$

The necessary optimal conditions [1], [9].

$$\left(H_{j_-} + \frac{\partial \varphi}{\partial t_{j_-}} + \mu^t \frac{\partial \vec{\chi}}{\partial t_{j_-}} \right) \delta t_{j_-} = 0 \quad (15)$$

$$\left(H_{j_+} - \frac{\partial \varphi}{\partial t_{j_+}} - \mu^t \frac{\partial \vec{\chi}}{\partial t_{j_+}} \right) \delta t_{j_+} = 0 \quad (16)$$

$$\left(\lambda_{j_-}^t - \frac{\partial \varphi}{\partial \vec{x}_{j_-}} - \mu^t \frac{\partial \vec{\chi}}{\partial \vec{x}_{j_-}} \right) \delta \vec{x}_{j_-} = 0 \quad (17)$$

$$\left(\lambda_{j_+}^t + \frac{\partial \varphi}{\partial \vec{x}_{j_+}} + \mu^t \frac{\partial \vec{\chi}}{\partial \vec{x}_{j_+}} \right) \delta \vec{x}_{j_+} = 0 \quad (18)$$

Where:

$\vec{\chi}$: the vector collecting the constraining boundary conditions (see eq. 15 - 18)

$\varphi = m_f$

At the initial point:

1. $\vec{r}_0 = \vec{r}_{\oplus}$;
2. $m_o = 1 - bV_{\infty} - cV_{\infty}^2$
3. $(\vec{v}_0 - \vec{v}_{\oplus})^2 = \vec{v}_{\infty}^2$;
4. Equations 16 and 18 provide optimal control with λ_{r_0} and T_{r_0} free;
5. the necessary condition optimal of the state is $\vec{\lambda}_{v_0}$ (primer vector) be parallel to the hyperbolic velocity;

At flyby [2]:

1. the equations (15 and 16) are used to obtain the transversality conditions, that implicates in determining the arc time used;
2. at the equations (17 and 18) the $\vec{\lambda}_{v_i}$ is parallel to the hyperbolic velocity, before and after of free flyby maneuver; the magnitude is continuous;
3. the states of Hamiltonian remain continuous through the flyby maneuvers;
4. when the minimum height constraint of the flyby is requested, a condition on the velocity turn angle is added (Eq. 2 and 3).

At the final point:

1. $\vec{\lambda}_{v_f}$ is parallel to the hyperbolic velocity, $\vec{\lambda}_{r_f}$ is parallel to the radius and $\vec{\lambda}_{r_f}^t \vec{v}_f + \vec{\lambda}_{v_f}^t \vec{g} = 0$;
2. the final values of $\vec{\lambda}_{m_f}$ and H_f depends on the control model that was considered in the maneuver;
3. the adjoint variable $\vec{\lambda}_v$ is zero during the whole trajectory.

4. MISSION ASTEROID 1989UQ

The following types of missions had been simulated:

1. without flyby;
2. Earth Gravity Assisted - EGA mission
3. Earth and Venus Gravity Assisted - EVGA mission

Using the optimization procedure we can find optimal trajectories, with the maximization of the spacecraft final mass (i.e., minimum fuel consumption). These trajectories depend on the mission objectives, for example, the performance depends on the mission time length. It is possible to reduce the time with some more spend of propellant.

The Keplerian elements of the asteroid 1989UQ, are shown:

Table 1 – Keplerian Elements.

Name	1989UQ
Epoch	54200
a	0.915249
e	0.264832
i	1.29152
Ω	178.2965
ω	15.0241
M	189.172
r_a	1.157636
r_p	0.672861

4.1 NUMERICAL ANALYSIS WITH PPS1350 (ESA)

The characteristics of the spacecraft propulsion system that have been assumed are [9]:

1. the mass of the spacecraft with an altitude of 200 km in circular LEO is 2133.3 Kg;
2. specific impulse $I_s = 1550s$;
3. specific energy $\epsilon = 0.06$;
4. $T = 2 \cdot 70mN$ (thruster PPS 1350 used for the SMART-1 mission to the moon);
5. nominal thruster $T_o = 1 UA$;
6. The time: $time = 0$ corresponds to the date 01/01/2000.

The necessary optimal condition were formulated in agreement with the problem; the bang-bang control was used in the formularization with limited power and constraint in the time of flight.

Table 2 – Simulation with PPS 1350 (ESA)

Asteroid 1989UQ				
PPS 1350 ($I_s = 1550s$) (2 x 70mN)				
Duration ΔT (days)	Δm	Data	ΔV_{el} (Electric) (km/s)	V_∞ (km/s)
<i>Flyby: 0</i>				
893.48	0.7652	Departure: 01/10/2017 Arrival: 13/03/2020	4.06805051	1.25579628
<i>Flyby: 1 – Earth (EGA)</i>				
920.09	0.8494	Departure: 11/07/2025 Flyby - Earth: 30/09/2026 Arrival: 17/01/2028	2.481256546	0.800492443
<i>Flyby: 2 - Earth – Venus (EVGA)</i>				
1164.92	0.8792	Departure: 25/06/2017 Flyby - Earth: 19/09/2018 Flyby - Venus: 05/03/2019 Arrival: 02/09/2020	1.956977088	0.694198512

4.2 NUMERICAL ANALYSIS WITH PHALL 1 (UNB)

The researchers of the Plasma Laboratory of the Physics Institute of the Brasilia University (UNB), since 2002, pledge in the study and development of a propellant that uses a plasma propulsion system produced by current Hall, based on Stationary Plasma Thrusters (SPT). In this project uses permanent magnets with generating the magnetic field, reducing the electricity consumption.

The characteristics of the spacecraft propulsion system are:

1. the mass of the spacecraft with an altitude of 200 km in circular LEO is 2133.3 Kg;
2. specific impulse $I_s = 1607s$;
3. specific energy $\epsilon = 0.06$;
4. $T = 2 \cdot 126mN$ (thruster Phall 1 - UNB);
5. nominal thruster $T_o = 1 UA$;
6. The time: $time = 0$ corresponds to the date 01/01/2000.

Diverse missions can be implemented with the optimization algorithm used in this work, of which the main ones are: transference with free time (to change to space vehicle orbit without restrictions to the necessary time the execution maneuver); “Rendezvous” (one desires that the space vehicle if finds and remains to the side of as a space vehicle); “Flyby” (desires to intercept one another celestial body, however without the objective to remain next); “Swing-By” (is used of a next ticket to a celestial body to gain or to lose energy, speed and angular moment), etc.

Been verified resulted better in comparison to the results gotten with PPS 1350 (Table 2 and 3), therefore, Phall 1 possess a bigger specific thrust and the thruster (t) is bigger in magnitude.

Table 3 - Simulation with Phall

Asteroid 1989UQ				
PHall (UNB) ($I_s = 1607s$) (2 x 126mN)				
Duration ΔT (days)	Δm	Data	ΔV_{el} (Electric) (km/s)	V_∞ (km/s)
<i>Flyby: 0</i>				
1305.5	0.7822	Departure: 01/10/2017 Arrival: 29/04/2021	3.870516065	0.986023774
<i>Flyby: 1 – Earth (EGA)</i>				
567.62	0.8244	Departure: 11/08/2024 Flyby - Earth: 10/11/2025 Arrival: 02/03/2026	3.04349697	0.844407717
<i>Flyby: 2 - Earth – Venus (EVGA)</i>				
1096.3	0.9177	Departure: 16/06/2017 Flyby - Earth: 20/09/2018 Flyby - Venus: 06/03/2019 Arrival: 16/06/2020	1.353277282	0.748891517

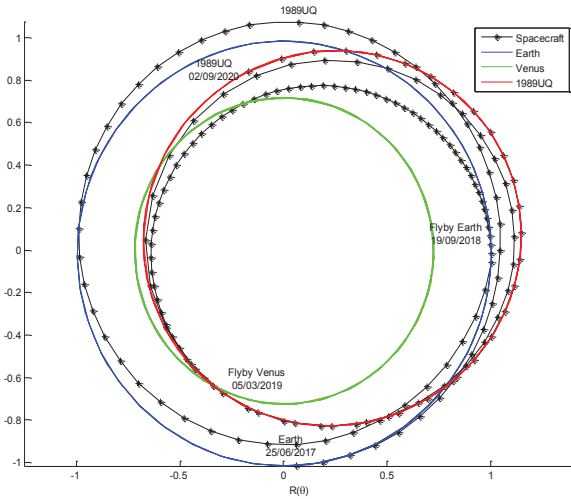


Figure 1 - Trajectory leaving from the Earth and arriving to the asteroid 1989UQ using EVGA maneuvers, PPS1350 (ESA).

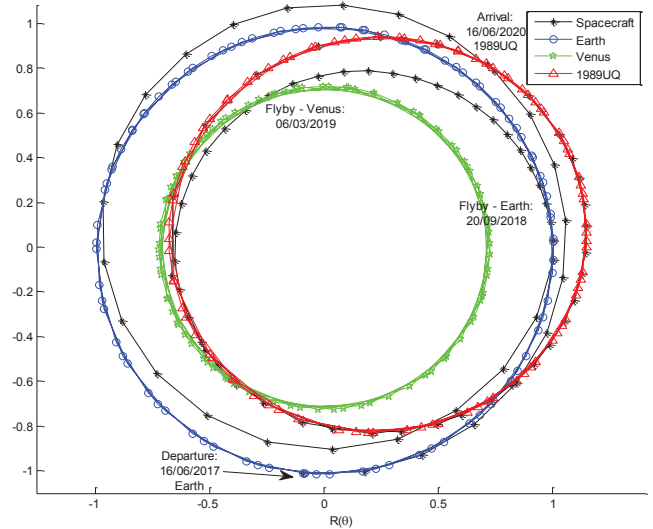


Figure 2- Trajectory leaving from the Earth and arriving to the asteroid 1989UQ using EVGA maneuvers, Phall 1 (UNB).

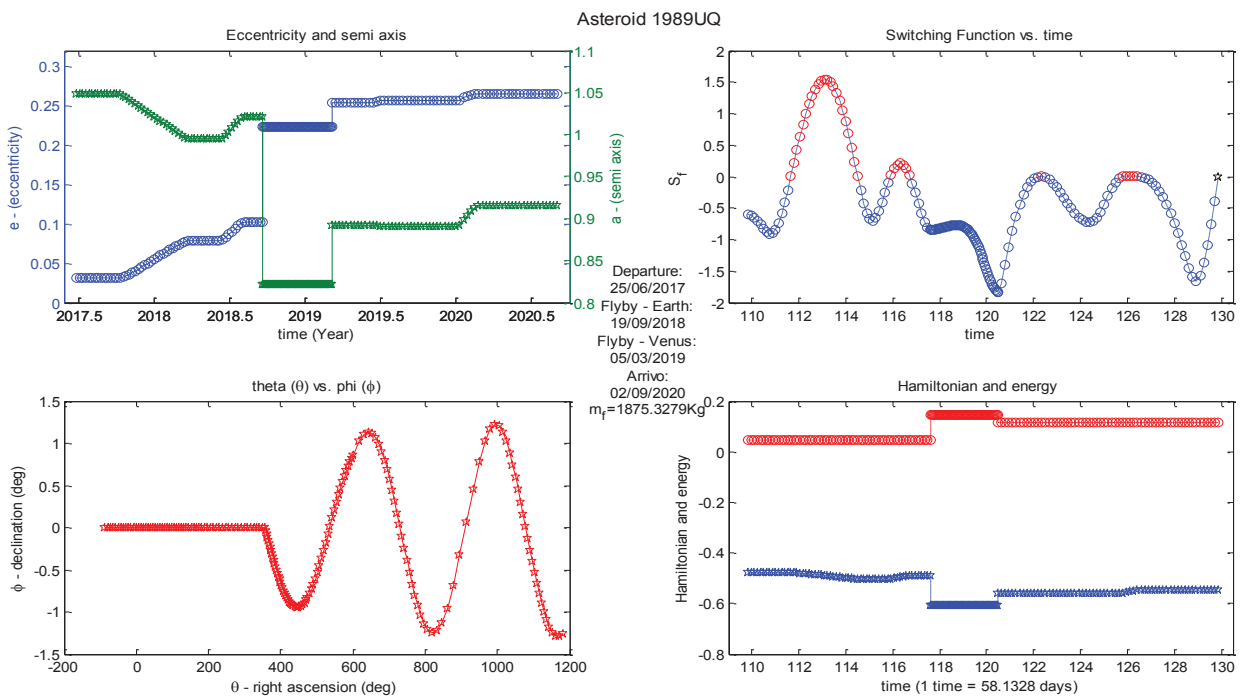


Figure 3 - Simulation with multiple flyby's: in the Earth and in Venus for the asteroid 1989UQ, with PPS1350.

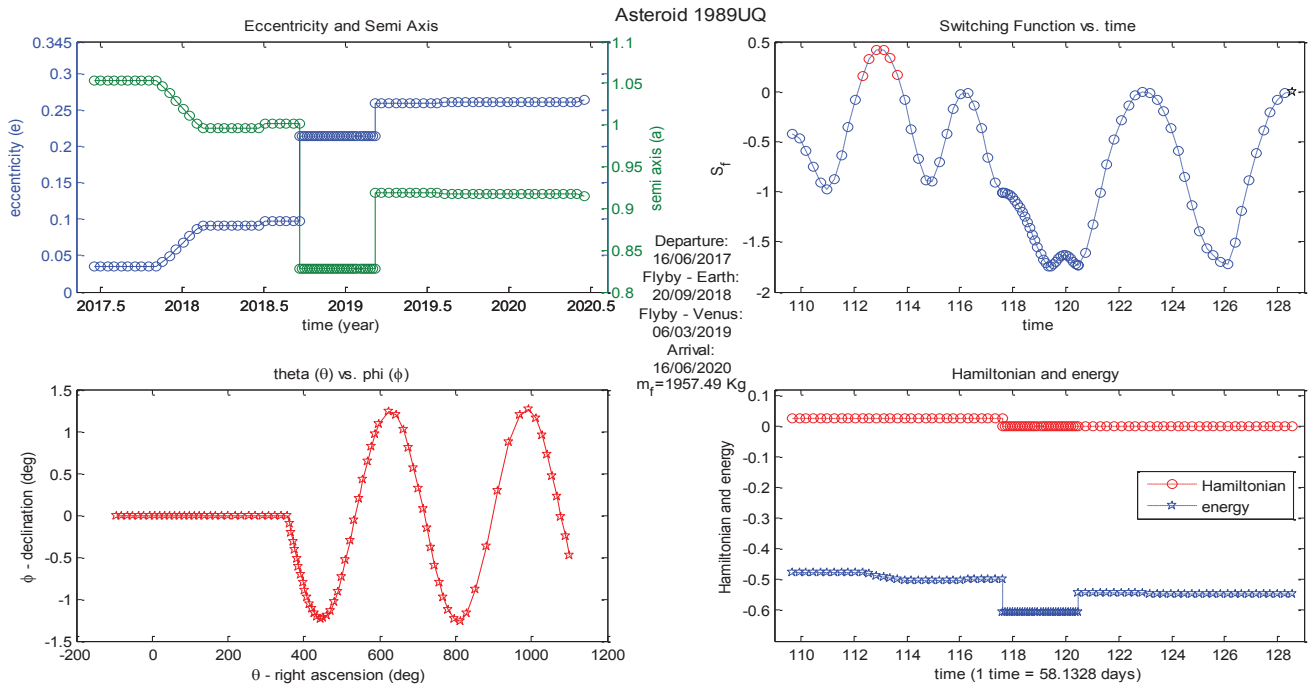


Figure 4 - Simulation with multiple flyby's: in the Earth and in Venus for the asteroid 1989UQ, with Phall 1.

In Figure 2 and Figure 4 is possible visualize the optimized orbit of the space vehicle leaving the Earth, making one year later one Earth flyby (EGA) and later one flyby in Venus (EVGA), and, arriving at the asteroid 1989UQ. The ΔT (time) total of the mission is of approximately 1165 days and Δm (final mass) it is 1875,6 kg.

The orbit of the asteroid 1989UQ crosses the orbit of the Earth, increasing the interest and importance of the study in relation to this asteroid, therefore asteroid of this type can cause damages to the planet or perturbation in the orbit of the terrestrial artificial satellites, because the proximity.

The Figures (2) and Figure (4) shown with the total time mission (ΔT) is of approximately 1096 days, the final mass useful (Δm) equal the 1957,7 kg, being optimum value for mass (Δm) of all the maneuvers for the asteroid 1989UQ. It is noticed in the commutation function (S_f) that a stretch of propulsion arc only exists and in remains of the trajectory, the thruster is off, stimulated for the gain gotten with the gravitational maneuvers (Flyby).

5. CONCLUSION

The search for the best initial parameters for a mission is facilitated if the transfer orbit with free time is optimized first. The ideal asteroids for EGA missions should possess low orbit energy, perihelion close to 1 UA, low inclination per EGA.

Indirect optimization methods based on optimal control theory supply accurate solutions. The use of Gravity

Assisted Maneuver (EGA, EMGA or EVGA) in this mission reduces the fuel consumption and the time of the maneuver, demonstrating that this important formulation is viable and useful.

Orbits with Phall 1 had been analyzed using gravity assisted maneuvers and verified resulted optimistically for the implantation of probes using this technology, also being able to use this formularization in the future missions that use launch vehicle that is in development/improvement (VLS-2, Brazil), which can inject in LEO (low earth orbit) a satellite medium sized, thereafter, use the solar electric propulsion (SEP) or nuclear (NEP) to dislocate the vehicle for desired orbits, maximizing them with the maneuver that use assisted gravity.

The present analysis favor a guess at the tentative solution as the Earth's positions as departure and flyby are a priori known. The ideal asteroid has perihelion radius which is close to 1AU, a low-energy orbit and low inclination with relation to the ecliptical axis.

The performance parameters of Phall are competitive with known electromagnet Hall thrusters found on the literature.

The fuel consumption for a mission with multiples flyby's follows the criterion of the asteroid orbit.

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