

# Development of uninterrupted correlations of water vapor pressure for heat pipes in a wide temperature range

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**Abstract.** *The performance and operation limits of heat pipes purely depend on thermo-physical properties. Any mathematical model must have adequate treatment of fluid properties that critically depend on temperature. Applications of heat pipes in satellites usually call for a very wide temperature range, therefore, it is very important to have correct and precise property correlations for the entire temperature two-phase range at saturation conditions, as well as out of the range. Simulation of HP (heat pipes) transient modes needs smoothed property correlations without interruptions of value and derivatives; tables and piece-wise linear interpolations between tabulated magnitudes are not acceptable. In this paper specific approximations were developed for water vapor pressure, which covers solid, two-phase, and super-critical temperature ranges. The results first time are presented in the format of a universal algorithm of a generalized function of dimensionless pressure versus dimensionless temperature. Such a function can be easily coded with any programming language and inserted into a heat pipe mathematical model.*

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**Keyword:** Triple Point, Critical Point, Interfacing, Numeric Simulation, Heat Pipes.

## 1. Introduction

Only in an ideal world, phenoms related to heat transfer can be disregarded, in the real world instead, these phenoms compel the project to consider it because if the system was dimensioned using only the ideal idea, this mission will fade in to fail. Any electronic equipment produces heat as a result of the second law of thermodynamics. The thermal system controls the heat in the satellite system, and usually, the design solution includes heat pipes, especially for big satellites. (SHUKLA, 2015) made an excellent explanation of the concept of heat pipes: “A typical heat pipe consists of a sealed pipe or tube made of a material that is compatible with the working fluid such as Copper for water heat pipes, or Aluminum for ammonia heat pipes. It is a simple construction that makes a heat pipe to allow high heat transfer rates over considerable distances, with minimum temperature drops...”.

The heat pipe must retain vapor-liquid equilibrium with the saturated liquid and its vapor. The saturated liquid vaporizes and flows to the condenser compartment, where it is cooled

and turned back to a saturated liquid. In a standard heat pipe, the condensed is returned to the evaporator using a wick structure applying a capillarity on the liquid phase of the working fluid.

Heat pipes are important and effective components of spacecraft thermal control systems and terrestrial applications. The performance of such devices depends on fluid properties at saturation conditions. Usually, such devices cannot operate outside their temperature limits. When the temperature is below the triple point, solidification or freezing occurs in working fluids; when the temperature is above the critical conditions, two phases cannot coexist anymore. However, the heat pipe must be able to be started from these states.

Any mathematical model must have adequate treatment of such properties that critically depend on temperature. Applications of heat pipes in satellites usually call for a very wide operational temperature range, therefore, it is very important to have correct and precise property correlations for the entire temperature range which include solid, two-phase, and super-critical zones. Simulation of HP (heat pipes) transient modes needs smoothed property correlations without interruptions of value and derivatives; tables and piece-wise linear interpolations between tabulated magnitudes are not acceptable. The most difficult problem arises in the simulation of specific transient HP modes, like start-up from solid state (high-temperature and freezable HPs) and start-up from super-critical states in cryogenics HPs.

## 2. Methodology

Usually, a polynomial approximation does not reflect the physical nature of the property behavior and only provides a good approximation within a limited temperature range.

Usually, this temperature range of polynomial approximations is narrower than the entire operating temperature range to keep the necessary precision. When the temperature magnitude jumps out of the approximation range, the values of a property calculated by the polynomial approximation may run away from the physical sense and may get absurd magnitudes.

Using such a polynomial approximation, the iteration process in a numerical algorithm may temporarily get out of the expected temperature range leading to numerical instability, and the algorithm failure.

To avoid this drawback, it is suggested for any thermo-physical property  $y$ , the approximation configuration of a function of temperature shall follow the following format whenever possible:

$$y(p, q, T) = Y(p, T) + \Delta y(q, T) \quad (1)$$

Where the first term – a basic correlation, which reflects the main physical process, and second term – auxiliary correction approximation.

For the auxiliary approximation term  $\Delta y(T)$ , the function used in the approximation must have asymptotical stability over the entire temperature range

The correlation for the main  $Y(T)$  and auxiliary  $\Delta y(T)$  terms of the final approximation include parameters (vectors  $p$  and  $q$ ) those values shall be obtained from the conditions of the best fitting of known tabulated data  $\hat{Y}(T_i)$  for tabulated temperatures  $T_i$ . We denote such parameter vectors as  $p$  and  $q$ , and can rewrite our approximation in a more detailed format:

These components of vectors  $p$  and  $q$  are obtained by minimization of deviations from tabulated data. The least-square technique may work fine. In mathematic terms it can be expressed as follows.

$$\min_{p,q} \sum_{i=1}^N \left( Y(p,q,T_i) - \hat{Y}(T_i) \right)^2 \quad (2)$$

where  $N$  – number of points available in a table of the given thermo-physical property. Obviously, to perform this stage, an empirical preliminary study shall be conducted to find an appropriate analytical format for the auxiliary term  $\Delta y(q,T)$ .

For generality, we will use dimensionless properties and temperature:

$$\tau = \frac{T - T_3}{T_{cr} - T_3} \quad p = \frac{P - P_3}{P_{cr} - P_3} \quad (3)$$

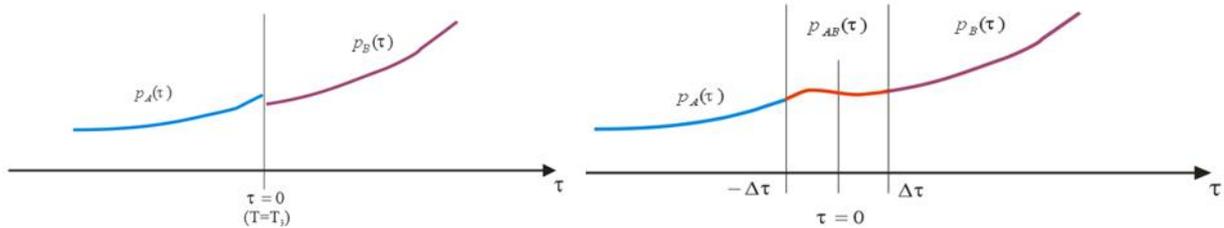
Wherefore, for two-phase zone, the values for both  $\tau$  and  $p$  may vary from 0 to 1. For sublimation,  $\tau$  and  $p$  are negative, and for the one-phase zone (i.e., gas), the values are above 1. Such approach will help to elaborate universal correlations with close coefficients for different working fluids. We denote these zones A, B e C,

To reach the smoothness needed in mathematical equations, the interfacing technique is used to connect the pressure equations in the interfacing points  $\tau=0$  and  $\tau=1$ .

Initially, the interfacing is applied in equations close to the triple point ( $\tau = 0$ ).

For example, good approximations are available for the sublimation zone, at  $\tau < 0$ , ( $p_A(\tau)$ ) and another – for two-phase zone ( $p_B(\tau)$ ).

Usually, at  $\tau=0$  ( and similar at  $\tau=1$ ) we have interruption Figure 1 for the magnitude and for derivatives from two sides: from the left and from the right  $p_A(0) \neq p_B(0)$  and  $p'_A(0) \neq p'_B(0)$ :



**Figure 1 – Interfacing for pressure at the division between sublimation and two-phase regions**

Such an interruption is not acceptable for HP numerical modeling in a wide range of temperatures. To resolve this problem, the idea is to introduce a small interface zone (AB) around  $T_3$  temperature to smooth these interruptions.

This  $\Delta\tau$  corresponds to a small value of temperature, for example,  $1^\circ\text{C}$ . The approximation in this interfacing zone,  $p_{AB}(\tau)$ , must eliminate and smooth these interruptions. To do so, we can establish additional constraints on this approximation (conditions of smoothing in interfacing zone), considering that we already have the approximations  $p_A(\tau)$  and  $p_B(\tau)$  and their derivatives:

$$\begin{cases} p_{AB}(-\Delta\tau) = p_A(-\Delta\tau) \\ p'_{AB}(-\Delta\tau) = p'_A(-\Delta\tau) \\ p_{AB}(\Delta\tau) = p_B(\Delta\tau) \\ p'_{AB}(\Delta\tau) = p'_B(\Delta\tau) \end{cases} \quad (4)$$

Therefore, we have 4 conditions for the interfacing. In this case a polynomial function of 3rd order can be pretty used:

$$p_{AB}(z) = a + bz + cz^2 + dz^3 \quad (5)$$

After the substitution of this approximation into the condition's equation (4),

we have a system of 4 algebraic equations of 4 unknowns a,b,c,d, which can be resolved by any method.

A similar approach was used for the interfacing of approximations around critical temperature  $T_{cr}$  ( $\tau = 1$ ).

### 3. Results

#### 3.1 Sublimation zone

Thermo-physical properties of water were studied widely and there are many published data and tables available for the using (IAPWS, 1995). Therefore, it is natural that this analysis is initiated by water. All analyzes will be treated in the same way in terms of temperature. Initially, the analysis will be done for  $\tau < 0$  starting at  $0 < \tau < 1$  and ending at  $\tau > 1$ .

For the sublimation zone ( $\tau < 0$ ), the best approximation which fits available table data is an exponential function.

$$p_A(\tau) = ae^{b\tau} + c \quad (6)$$

**Table 1 - Coefficients of exponential approximation for  $\tau < 0$**

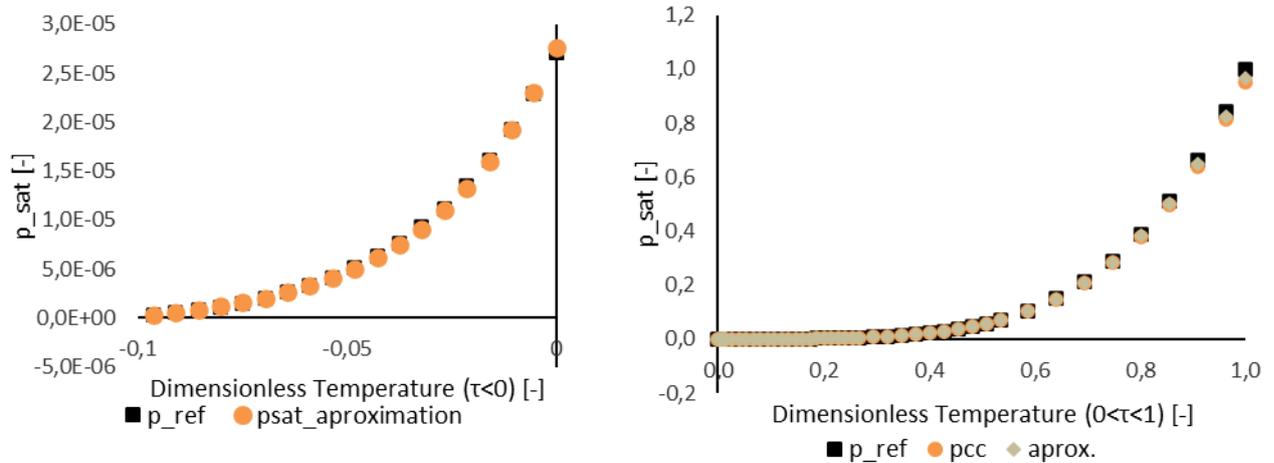
	a	b	c
$p_A(\tau)$	2,85E-05	3,26E+01	-9,00E-07

The resulting approximation is shown in Figure 2, on the left.

### 3.2 Two-phase zone

For  $0 < \tau < 1$ , the approximation of the vapor saturation pressure is based on an already known variation, the Clapeyron-Clausius pressure used in many studies, as (KOUTSOYIANNIS, 2012) did when aligning the theory with the empirical data.

First, we perform analysis with temperature as a dimensionless variable keeping pressure as a dimensional variable. This was done using the approach discussed in the previous chapter, the results can be seen below in Figure 2, on the right.



**Figure 2- Correlation between saturation pressure and dimensionless Temperature for water below the triple point ( $\tau < 0$ ) and above it ( $0 < \tau < 1$ )**

Then, the approximation was performed for dimensionless variables following the proposed approach (Eq.1). The results are shown in the equation below, in equations (7) and (8). It is worth mentioning that, this approximation is based on a classic of the Clapeyron-Clausius equation,  $P_{CC}()$ , with an added polynomial approximation of 3rd order for refinement.

$$p_{CC} = (p_0 + p_3) \text{Exp} \left[ \frac{\lambda_0}{R(T_3 + \tau_0(T_{cr} - T_3))} \left( 1 - \frac{T_3 + \tau_0(T_{cr} - T_3)}{T_3 + \tau(T_{cr} - T_3)} \right) \right] - p_3 \quad (7)$$

$$p_B(\tau) = p_{cc} + a\tau^3 - b\tau^2 - c\tau - d \quad (8)$$

**Table 2 – Coefficients results of  $0 < \tau < 1$  with dimensionless pressure and temperature**

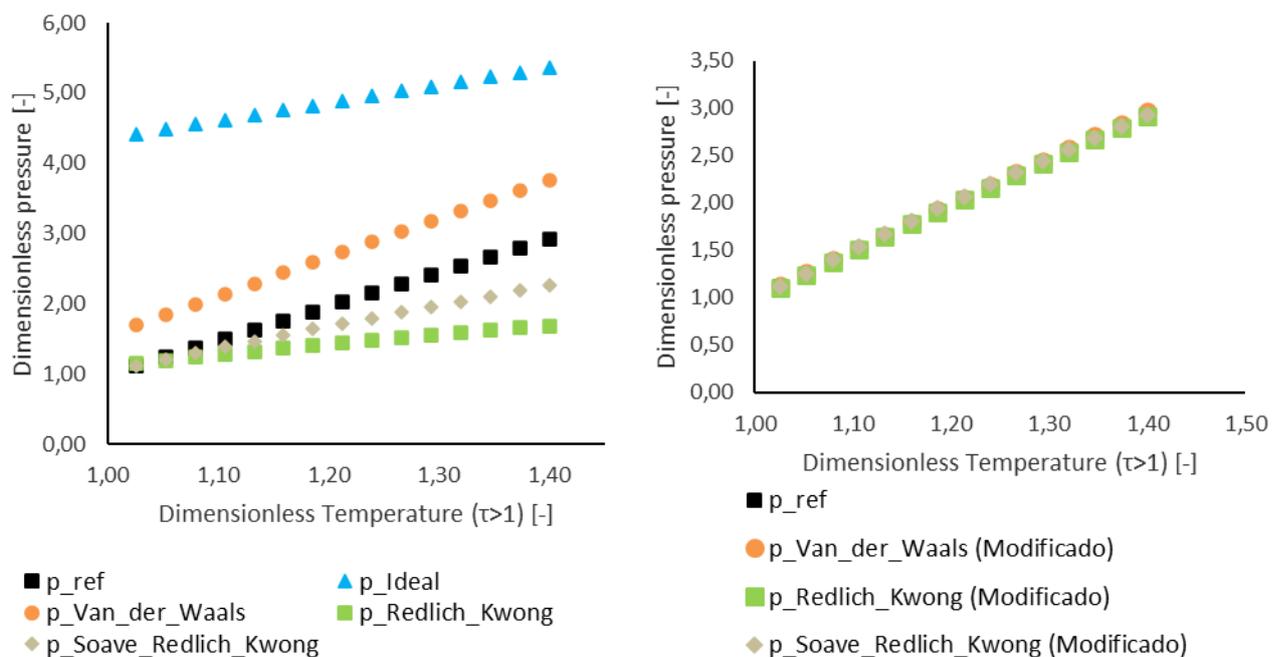
	<b>a</b>	<b>b</b>	<b>c</b>	<b>d</b>	<b>T<sub>0</sub> [°C]</b>
$p_B(\tau)$	2.3E-03	-1.02E-02	-5E-05	-3E-05	95

### 3.3 Supercritical zone

To obtain precision approximation the pressure above the critical point cannot be treated through a well-known ideal gas law. We will use a real gas approach based on the Van der Waals equation, Redlich-Kwong equation, and Soave-Redlich-Kwong equation. The better option was selected to use for the approximation and interfacing at  $\tau=1$ .

The heat pipe has a peculiarity. The total density of working fluid, considering liquid and vapor phases together, is fixed and defined by the working fluid amount charged into the heat pipe during HP fabrication. Then the HP container is sealed hermetically, and the mass of working fluid does not change. Knowing the internal volume of heat pipe, such total density can be easily calculated:

Therefore, the author chose more than one fluid density for the analysis; which varies from  $332 \text{ kg/m}^3$  (critical density) up to  $350 \text{ kg/m}^3$  but for this paper only the critical density will be analysed. The charts presented in Figure 3 can be expressed by the Van der Waals equation (9), Redlich-Kwong equation (12), and Soave equation (15), (SMITH; PETERS; INOMATA, 2013; MARKOČIĆ; KNEZ, 2016).



**Figure 3 - Correlation between Pressure and Dimensionless Temperature for water above the critical point and with a density equal to 322 kg/m<sup>3</sup> and suggested modifications**

Below the real-gas equations were modified to achieve dimensionless parameters. Van der Waals equation can be expressed as follows:

$$p = \frac{R \cdot (T_3 + \tau(T_{cr} - T_3))}{\left(\frac{M}{\rho_{Crü}} - b\right)(P_{CR} - P_3)} - \frac{a}{\left(\frac{M}{\rho_{Crü}}\right)^2 (P_{CR} - P_3)} - \frac{P_3}{P_{CR} - P_3} \quad (9)$$

$$a(\tau) = \frac{(a_1(T_3 + \tau(T_{cr} - T_3)))^2 + b_1(T_3 + \tau(T_{cr} - T_3)) + c_1) \cdot R^2 \cdot T_{CR}^2}{P_{CR}} \quad (10)$$

$$b(\tau) = \frac{R \cdot T_{CR}}{(a_1(T_3 + \tau(T_{cr} - T_3)))^2 + b_1(T_3 + \tau(T_{cr} - T_3)) + c_1) P_{CR}} \quad (11)$$

Redlich-Kwong equation:

$$p = \frac{R \cdot (T_3 + \tau(T_{CR} - T_3))}{\left(\frac{M}{\rho_{Crü}} - b\right)(P_{CR} - P_3)} - \frac{a}{\left(\frac{M}{\rho_{Crü}}\right) \left(\frac{M}{\rho_{Crü}} + b\right) (P_{CR} - P_3) T^{1/2}} \quad (12)$$

$$a(\tau) = \frac{R^2 \cdot T_{CR}^{5/2}}{(a_1(T_3 + \tau(T_{cr} - T_3)))^2 + b_1(T_3 + \tau(T_{cr} - T_3)) + c_1) P_{CR}} \quad (13)$$

$$b(\tau) = \frac{R \cdot T_{CR}}{(a_1(T_3 + \tau(T_{cr} - T_3)))^2 + b_1(T_3 + \tau(T_{cr} - T_3)) + c_1) P_{CR}} \quad (14)$$

Soave-Redlich-Kwong equation:

$$p = \frac{R \cdot (T_3 + \tau(T_{CR} - T_3))}{\left(\frac{M}{\rho_{Crü}} - b\right)(P_{CR} - P_3)} - \frac{a}{\left(\frac{M}{\rho_{Crü}}\right) \left(\frac{M}{\rho_{Crü}} + b\right) (P_{CR} - P_3)} - \frac{P_3}{(P_{CR} - P_3)} \quad (15)$$

$$a(\tau) = \frac{R^2 \cdot T_{CR}^2}{(a_1(T_3 + \tau(T_{cr} - T_3)))^2 + b_1(T_3 + \tau(T_{cr} - T_3)) + c_1) P_{CR}} \left(1 + m \left(1 - T_R^{1/2}\right)\right)^2 \quad (16)$$

$$b(\tau) = \frac{R \cdot T_{CR}}{(a_1(T_3 + \tau(T_{cr} - T_3)))^2 + b_1(T_3 + \tau(T_{cr} - T_3)) + c_1) P_{CR}} \quad (17)$$

$$m = 0.480 + 1.574\omega - 0.176\omega^2 \quad (18)$$

$$\omega = \log_{10} \left( \frac{P_{CR}}{P_{VP}} \right) \quad (19)$$

$$T_R = \frac{T}{T_{CR} + 8} \quad (20)$$

Those coefficients expressed in variable “a” and “b” can be expressed in the table below.

**Table 3 - Coefficients “a” and “b” expressed by polynomial coefficients**

		<b>a1</b>	<b>a1 <math>\rho= 322</math> kg/m<sup>3</sup></b>	<b>b1</b>	<b>b1 <math>\rho= 322</math> kg/m<sup>3</sup></b>	<b>c1</b>	<b>c1 <math>\rho= 322</math> kg/m<sup>3</sup></b>
Van der Waals	<b>a</b>	-	-6,00E-08	-	9,00E-05	4,22E+04	3,97E+02
	<b>b</b>	-	-	-	7,00E-04	8.00	7.81
Redlich-Kwong	<b>a</b>	-	-	-	2.4E-03	23.393	7,26E+02
	<b>b</b>	-	-	-	-9.6E-3	11.542	1,79E+05
Soave-Redlich-Kwong	<b>a</b>	-	-	-	1.3E-02	23.393	1.485
	<b>b</b>	-	-	-	-5.8E-3	11.542	1,54E+04

### 3.3 Final results in the form of universal algorithm and dimensionless variables

The results first time are presented in the format of a universal algorithm of a generalized function of dimensionless pressure versus dimensionless temperature. Such a function can be easily coded in any programming language and inserted into any mathematical model of heat pipes. This function is shown in equation (21) below.

$$p(\tau) = \begin{cases} \text{if } (\tau \leq -\Delta z) \text{ then } p_A(\tau) = ae^{b\tau} + c \\ \text{if } (-\Delta z < \tau < \Delta z) \text{ then } p_{AB}(\tau) = a_{AB} + b_{AB}\tau + c_{AB}\tau^2 + d_{AB}\tau^3 \\ \text{if } (\Delta z \leq \tau \leq (1-\Delta z)) \text{ then} \\ p_B(\tau) = (p_0 + p_3) \text{Exp} \left[ \frac{\lambda_0}{R(T_3 + \tau_0(T_{cr} - T_3))} \left( 1 - \frac{T_3 + \tau_0(T_{cr} - T_3)}{T_3 + \tau(T_{cr} - T_3)} \right) \right] - p_3 \\ \quad + a\tau^3 - b\tau^2 - c\tau - d \\ \text{if } ((1-\Delta z) < \tau < (1+\Delta z)) \text{ then } p_{BC}(\tau) = a_{BC} + b_{BC}\tau + c_{BC}\tau^2 + d_{BC}\tau^3 \\ \text{if } (\tau \geq (1+\Delta z)) \text{ then} \\ p_C(\tau) = \frac{R \cdot (T_3 + \tau(T_{CR} - T_3))}{\left( \frac{M}{\rho_{Cr\ddot{u}}} - b \right) (P_{CR} - P_3)} - \frac{a}{\left( \frac{M}{\rho_{Cr\ddot{u}}} \left( \frac{M}{\rho_{Cr\ddot{u}}} + b \right) \right) (P_{CR} - P_3)} - p_3 \end{cases} \quad (21)$$

$$\begin{cases} a_{AB} = 1.19269 \cdot 10^{-5} \\ b_{AB} = -7.59439 \cdot 10^{-6} \\ c_{AB} = 1.0423 \cdot 10^{-2} \\ d_{AB} = 4.62858 \cdot 10^{-1} \end{cases} \begin{cases} a_{BC} = 1.47843 \cdot 10^3 \\ b_{BC} = -4.05843 \cdot 10^3 \\ c_{BC} = 3.67735 \cdot 10^3 \\ d_{BC} = -1.09638 \cdot 10^3 \end{cases} \quad (22)$$

The coefficients a,b,c for sublimation and super-critical are already shown in Tables 1-3.

## Conclusion

This work provides approximations for water-saturated pressure from low temperature (below triple point) to high temperature (above critical point). This pressure is the main parameter in any mathematical model of heat pipes and other two-phase heat transfer devices. Different types of approximations were combined in the equations using the interfacing interrupted method, in other words, the results reached until here give a continuous property result for values as well as for derivatives. The results first time are presented in the format of a universal algorithm of a generalized function. Such a function can be easily coded with any programming language and inserted into any mathematical model of heat pipes. For future work more fluids can be add in this algorithm to reach a hole group of interesting work fluids in aerospace engineering.

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## Annex

$T$	Temperature [°C]
$T_3$	Triple point temperature [°C]
$T_{CR}$	Critical temperature [°C]
$P$	Pressure [bar]
$p_{cc}$	Clausius-Clapeyron Saturation Pressure Equation dimensionless
$p_0$	Reference pressure dimensionless
$P_3$	Triple point pressure [bar]
$p_3$	Triple point pressure dimensionless [bar]
$Y(T)$	Equation using temperature as variable
$\tau$	Temperature Dimensionless
$\Delta \tau$	Temperature Dimensionless Variation
$\tau_0$	Reference Temperature Dimensionless
$p_A(\tau)$	Saturated pressure equation $\tau < 0$
$p'_A(\tau)$	Derivation of Saturated pressure equation $\tau < 0$
$p_B(\tau)$	Saturated pressure equation $0 < \tau < 1$
$p'_b(\tau)$	Derivation of Saturated pressure equation $0 < \tau < 1$
$p_C(\tau)$	Pressure equation $\tau > 1$
$p'_c(\tau)$	Derivation of pressure equation $\tau > 1$
$p_{AB}(\tau)$	Interfacing equation linking saturated pressure from $\tau < 0$ to $0 < \tau < 1$
$p'_{AB}(\tau)$	Derivation of Interfacing equation linking saturated pressure from $\tau < 0$ to $0 < \tau < 1$

- $p_{bc}(\tau)$  Interfacing equation linking saturated pressure from  $0 < \tau < 1$  to pressure  $\tau > 1$
- $\dot{p}_{bc}(\tau)$  Derivation of Interfacing equation linking saturated pressure from  $0 < \tau < 1$  to pressure  $\tau > 1$
- $a$  Variable
- $b$  Variable
- $c$  Variable
- $d$  Variable