LISA sensitivities to gravitational waves from relativistic metric theories of gravity

Massimo Tinto*

Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California 91109, USA

Márcio Eduardo da Silva Alves†

*INPE, Instituto Nacional de Pesquisas Espaciais, Divisão de Astrofísica, Avenida dos Astronautas 1758,
São José dos Campos, 12227-010 SP, Brazil*
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The direct observation of gravitational waves will provide a unique tool for probing the dynamical properties of highly compact astrophysical objects, mapping ultrarelativistic regions of space-time, and testing Einstein's general theory of relativity. LISA (Laser Interferometer Space Antenna), a joint National Aeronautics and Space Administration and European Space Agency mission to be launched in the next decade, will perform these scientific tasks by detecting and studying low-frequency cosmic gravitational waves through their influence on the phases of six modulated laser beams exchanged between three remote spacecraft. By directly measuring the polarization components of the waves LISA will detect, we will be able to test Einstein's theory of relativity with good sensitivity. Since a gravitational wave signal predicted by the most general relativistic metric theory of gravity accounts for *six* polarization modes (the usual two Einstein's tensor polarizations as well as two vector and two scalar wave components), we have derived the LISA time-delay interferometric responses and estimated their sensitivities to vector- and scalar-type waves. We find that (i) at frequencies larger than roughly the inverse of the one-way light time ($\approx 6 \times 10^{-2}$ Hz), LISA is more than ten times sensitive to scalar-longitudinal and vector signals than to tensor and scalar-transverse waves, and (ii) in the low part of its frequency band is equally sensitive to tensor and vector waves and somewhat less sensitive to scalar signals.

I. INTRODUCTION

The direct detection of gravitational waves will represent one of the greatest triumphs of experimental physics of this century, and provide us with a new observational tool for obtaining better and deeper understanding about their sources.

Several experimental efforts have been underway for years, both on the ground and in space [1–6], and only recently kilometer-size ground-based interferometers have been able to identify the most stringent upper limits to date for the amplitudes of the radiation expected from several classes of sources. Although an unambiguous detection has not been declared with present-generation instruments, next-generation Earth-based interferometers and pulsar-timing experiments, as well as the LISA (Laser Interferometric Space Antenna) mission [7] are expected to achieve this goal.

LISA, jointly proposed to the National Aeronautics and Space Administration (NASA) and the European Space Agency (ESA), is expected to be flown sometime in the next decade. Its goal is to detect and study gravitational waves (GW) in the millihertz frequency band. It will use coherent laser beams exchanged between three identical spacecraft forming a giant (almost) equilateral triangle

of side 5×10^6 kilometers. By monitoring the relative frequency changes of the light beams exchanged between the spacecraft, it will extract the information about the gravitational waves it will observe at unprecedented sensitivities [7]. The astrophysical sources that LISA is expected to observe within its operational frequency band ($10^{-4} -$ 1 Hz) will be very large in number, including galactic and extragalactic coalescing binary systems containing white dwarfs and neutron stars, extragalactic supermassive black-hole coalescing binaries, and a stochastic gravitational wave background from the early universe [8,9].

The first unambiguous detection of a gravitational wave signal, whether performed on the ground or in space, will also allow us to test Einstein's general theory of relativity (GR) by measuring the polarization components of the detected signals [10,11]. Among all the proposed relativistic metric theories of gravity [12], GR is the most restrictive, allowing for only two of possible *six* different polarizations [13]. By asserting that the spin-2 ("tensor") polarizations are the only polarization components observed, we would make a powerful proof of the validity of Einstein's theory of relativity, while a clear observation of some other polarization modes would disqualify it. Corroboration of polarization measurements with estimates of the propagation speed of the observed gravitational wave signal will provide further insight into the nature of the observed radiation and result in the determination of the mass of the graviton. It should be emphasized, however,

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*Massimo.Tinto@jpl.nasa.gov
†Present address: Instituto de Ciências Exatas, Universidade Federal de Itajubá, 37500-903 Itajubá, MG, Brazil.
alvesmes@unifei.edu.br

that a propagation-speed measurement alone, consistent with a value equal to the speed of light, would not automatically rule-out other relativistic metric theories of gravity. This is because waves with helicity $s = 0$ can also propagate at light speed [14]. Gravitational waves with scalar polarization are predicted by the most common generalizations of GR such as scalar-tensor theories. Besides the classical example of Brans-Dicke theory [15], scalar-tensor theories result in the low-energy limit of string theory (see, e.g., [16]). Modifications of the Einstein-Hilbert action, which consider generic functions of the Ricci scalar in the Lagrangian [$f(R)$ theories], also predict "scalar" gravitational waves [17]. Vector modes, on the other hand, can appear in the so-called "quadratic gravity" formulations [17], and in the context of theories in which the graviton has a finite mass such as the Visser theory [18].

LISA will not be able to distinguish the propagation speeds of scalar (helicity $s = 0$) and vector (helicity $s = \pm 1$) polarizations from the speed of light (a result following, as we shall see below, from a combination of the existing stringent upper limits on the mass of the graviton [19–21] and the LISA observational bandwidth). However, it should be able to assess the polarizations of the observed gravitational wave signals. Since the accuracy by which LISA will distinguish one polarization from another will depend on their signal-to-noise ratios [22], in this paper we estimate the LISA sensitivities to scalar- and vector-polarized wave.

The paper is organized as follows. In Sec. II we derive the one-way Doppler response to a gravitational wave signal characterized by six polarizations ([two tensor (helicity $s = \pm 2$), two "vector" (helicity $s = \pm 1$), and two scalar (helicity $s = 0$)]. Since the resulting expression is equal *in form* to the one previously derived by Estabrook and Wahlquist [23] for tensor waves (i.e. for waves predicted by general relativity), we conclude that the responses of the various time-delay interferometric (TDI) combinations are also identical *in form* to those previously derived within the framework of GR [24]. Although the derivation of the response function of a Michelson interferometer to nontensor polarization modes has been considered in previous publications [10,25], the expression presented there was correct in the so-called "long-wave-length-limit," i.e. when the wavelength of the GW is much larger than the size of the detector. Since LISA will be sensitive, over most of its observational frequency band, to GWs of wavelength shorter than its linear size, we have derived the expressions of the LISA TDI responses to vector- and scalar-waves that are valid for any wavelength. In Sec. III we then compute the LISA sensitivities to vector and scalar waves. We find that (i) at frequencies larger than the inverse of the one-way light time ($\approx 6 \times 10^{-2}$ Hz) LISA is ten times more sensitive to scalar-longitudinal and vector signals than to tensor and scalar-transverse waves, while (ii) it is equally sensitive to tensor and vector

waves, and somewhat less sensitive to scalar signals in the low part of its frequency band. Although both these results might seem surprising at first, we show that they are consequence of the physical and geometrical properties of the vector and scalar waves and how they affect the frequency of the light beams exchanged by the three LISA spacecraft. Finally in Sec. IV we present a summary of the paper and our concluding remarks. Throughout the paper we will be using natural units ($c = G = h = 1$) except where mentioned otherwise.

II. DERIVATION OF THE ONE-WAY DOPPLER RESPONSE

In what follows we present the derivation of the "one-way" Doppler response to a gravitational wave signal predicted by the most general relativistic metric theory of gravity [13]. Although the result has already appeared in the literature [26], our derivation results into an expression that is compact and identical *in form* to that first obtained by Wahlquist [27] in the case of plane gravitational waves predicted by GR. This of course simplifies significantly the derivation of the LISA TDI responses as they turn out to be equal *in form* to those given in [24]. As in [23,27], our derivation is general and does not rely on any assumptions about the size of the wavelength of the radiation relative to the size of the detector.

Let us consider a space-time with the following metric:

$$ds^2 = -dt^2 + (\delta_{ij} + h_{ij}(vt - z))dx^i dx^j, \qquad |h_{ij}| \ll 1, \tag{1}$$

where the usual sum convention over repeated indices is assumed, Latin indices go from 1 to 3, and a plane-gravitational wave has been assumed, without loss of generality, to propagate along an arbitrary $+z$ direction. In Eq. (1) we have allowed the wave to travel at a finite speed (group velocity) $v < 1$ to account for a possible nonzero mass of the graviton. Working in the context of GR, it is well known that h_{ij} has 2 degrees of freedom representing gravitational waves (GWs) with helicity $s = \pm 2$, and $v = 1$. On the other hand, alternative relativistic metric theories of gravity allow for GWs with up to 6 degrees of freedom [13]. Therefore, in the most general case, h_{ij} can be represented in terms of six components (with corresponding six metric amplitudes) in the following form [13]:

$$h_{ij}(vt - z) = \sum_{r=1}^{6} \epsilon_{ij}^{(r)} h_{(r)}(vt - z), \tag{2}$$

where $\epsilon_{ij}^{(r)}$ are the six polarization tensors associated with the six waveforms of the gravitational wave signal. If we introduce a set of Cartesian orthogonal coordinates (x, y, z) associated with the wave, in which (x, y) are in the plane of the wave and z is along the direction of propagation of the wave and orthogonal to the (x, y) plane, the above six polarization tensors assume the following matrical form [13]:

$$[\epsilon^{(1)}]_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad [\epsilon^{(2)}]_{ij} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$[\epsilon^{(3)}]_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad [\epsilon^{(4)}]_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$[\epsilon^{(5)}]_{ij} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad [\epsilon^{(6)}]_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{3}$$

From the above expressions it is easy to verify that the tensors $\epsilon^{(a)}$, $a = 1, \ldots, 6$ are linearly independent and form an orthogonal basis. In our notation we have labeled 4 and 5 the usual $+$ and $\times$ polarizations, respectively (the tensor polarization waveforms). The vector polarizations ($s = \pm 1$) were labeled as 2 and 3, and finally the two scalar modes ($s = 0$) have been denoted with the labels 1 (for the longitudinal scalar polarization) and 6 (for the transversal scalar mode).

Following [23,28] we may notice that the space-time described by the metric of Eq. (1) allows for the following three Killing vectors:

$$K^\rho_{(1)} = \delta^\rho_x, \qquad K^\rho_{(2)} = \delta^\rho_y, \qquad K^\rho_{(3)} = \delta^\rho_z + \frac{\delta^\rho_t}{v}. \tag{4}$$

Generally speaking, in the weak field regime of a generic metric theory of gravity, GWs can travel with $v \leq 1$. In that case the linearized field equations are Klein-Gordon type, and the resulting group velocity is determined by the mass of the graviton, m. Remembering that $v = \partial\omega/\partial k$, and that the dispersion relation is equal to $k = \sqrt{\omega^2 - m^2}$, we find the following expression for v in terms of m and ω:

$$v(\omega) = \sqrt{1 - \left(\frac{m}{\omega}\right)^2}. \tag{5}$$

Since the operational frequency band of LISA will be within the range ($10^{-4} - 1$ Hz), by assuming presently known upper limits for the mass of the graviton from Eq. (5) we obtain the resulting values for the group velocity of these waves. After restoring physical units and taking $m < 10^{-59}$ g (the most stringent constraint to date obtained by requiring the derived dynamical properties of a galactic disk to be consistent with observations [21]), at 10^{-4} Hz we find a value for the group velocity v whose fractional difference, Δ_v, from the speed of light is equal to $\Delta_v = |v - c|/c \simeq 10^{-15}$. A less stringent value for the mass of the graviton equal to $m < 7.68 \times 10^{-55}$ g (obtained from solar-system dynamics observations [19]), results into a $\Delta_v \simeq 10^{-8}$. These considerations imply that, no matter whether we are conservative or not in our assumption about a likely upper limit for the graviton mass, LISA will not be able to resolve the propagation speeds of the different polarization components of the detected GWs

by resolving the time separations of their imprints in the TDI combinations [24]. For this reason, from now on, we will assume $v = 1$ in natural units, and rewrite the third Killing vector $\boldsymbol{K}_{(3)}$ as $K^\rho_{(3)} = \delta^\rho_z + \delta^\rho_t$.

Let us now consider the unit vector $\hat{n}$ along the Doppler link and oriented from spacecraft 1 to 2 (see Fig. 1). If we denote with (θ, ϕ) the usual polar angles, $\hat{n}$ assumes the following familiar form:

$$n^\rho = \sin\theta \cos\phi \, \delta^\rho_x + \sin\theta \sin\phi \, \delta^\rho_y + \cos\theta \, \delta^\rho_z. \tag{6}$$

Since the most general Killing vector of our metric can be written as a linear combination of the three Killing vectors above

$$K^\rho = a_1 \delta^\rho_x + a_2 \delta^\rho_y + a_3(\delta^\rho_z + \delta^\rho_t), \tag{7}$$

with a_1, a_2, a_3 constants, by comparing Eq. (6) and Eq. (7) we note that by taking $(a_1, a_2, a_3) = C\hat{n}$, Eq. (7) becomes

$$K^\rho = C[n^\rho + \hat{k} \cdot \hat{n} \delta^\rho_t], \tag{8}$$

where C is an arbitrary constant, and $\hat{k}$ is the unit vector along the direction of propagation of the wave (see Fig. 1).

If we now consider the 4-momentum of a photon transmitted by spacecraft 1, its analytic expression can be written in terms of the metric perturbation at spacecraft 1 in the following way [23,28]:

$$P_\rho = v_0(-\delta^t_\rho + n_\rho + \tfrac{1}{2}h_{\rho\xi}n^\xi), \tag{9}$$

where it is easy to see that the condition $P_\rho P^\rho = 0$ is fulfilled to first order in the metric perturbation h_{ij}. Since $P_\rho K^\rho = $ constant along the photon world line [23,28], we obtain the following relationship between the frequency of the photon emitted at spacecraft 1, v_0, and that received at spacecraft 2, v':
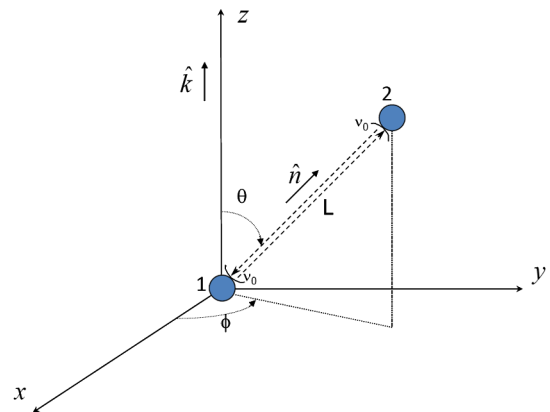

FIG. 1 (color online). A laser beam of nominal frequency v_0 is transmitted from spacecraft 1 to spacecraft 2 and simultaneously another beam from spacecraft 2 is transmitted back to 1. The gravitational wave train propagates along the z direction, and the two polar angles (θ, ϕ) describe the direction of propagation of the laser beams relative to the wave. See the text for a complete description.

$$P_\rho K^\rho = P'_\rho K'^\rho, \tag{10}$$

$$\nu_0(-\delta^t_\rho + n_\rho + \tfrac{1}{2}h_{\rho\xi}n^\xi)(n^\rho + \hat{k}\cdot\hat{n}\delta^\rho_t)$$
$$= \nu'(-\delta^t_\rho + n'_\rho + \tfrac{1}{2}h'_{\rho\xi}n'^\xi)(n'^\rho + \hat{k}\cdot\hat{n}'\delta^\rho_t). \tag{11}$$

If we now rewrite $n'^\rho = n^\rho + \delta n^\rho$, we may notice that, to first order in h_{ij}, $n^\rho\delta n_\rho = 0$ because $n'_\rho n^\rho = n_\rho n^\rho = 1$. This result, together with Eq. (11) above, allows us to derive the following expression for the ratio of the two frequencies ν' and ν_0:

$$\frac{\nu'}{\nu_0} = \frac{1 - \hat{k}\cdot\hat{n} + \tfrac{1}{2}n^\rho h_{\rho\xi}n^\xi}{1 - \hat{k}\cdot\hat{n} + \tfrac{1}{2}n^\rho h'_{\rho\xi}n^\xi}. \tag{12}$$

Finally, expanding the right-hand side of the last equation to first order in $h_{\rho\xi}$, we get the resulting expression for the one-way Doppler response, y:

$$y = (1 + \hat{k}\cdot\hat{n})(\Psi - \Psi'), \tag{13}$$

where $y(t) \equiv (\nu'(t) - \nu_0)/\nu_0$, and $\Psi(t)$ is equal to

$$\Psi(t) \equiv \frac{n^i h_{ij}(t)n^j}{2[1 - (\hat{k}\cdot\hat{n})^2]}. \tag{14}$$

By explicitly showing the time dependence of the various terms, $y(t)$ can be rewritten in the following form [24]:

$$y(t) = (1 + \hat{k}\cdot\hat{n})[\Psi(t - L) - \Psi(t - \hat{k}\cdot\hat{n}L)], \tag{15}$$

where L is the separation between the two spacecraft. Note that, in order to obtain the above expression, we have only assumed the time components of the metric perturbation, $h_{\rho t}$ to be equal to zero [13].

The expression for the one-way Doppler response measured on board spacecraft 1 at time t can be obtained from Eq. (15) by changing $\hat{n} \to -\hat{n}$, and further delaying the waveforms by $\hat{k}\cdot\hat{n}L$. The resulting one-way Doppler response, $y'(t)$, is equal to

$$y'(t) = (1 - \hat{k}\cdot\hat{n})[\Psi(t - (1 + \hat{k}\cdot\hat{n})L) - \Psi(t)]. \tag{16}$$

Since the above expressions of the one-way Doppler responses are identical *in form* to those valid for tensor waves [27], it follows that the various TDI combinations of the LISA six interspacecraft one-way Doppler measurements will also be identical *in form* to those derived in [24]. For these reasons they will not be given here, and we refer the reader to [24,29,30] for more details.

III. LISA SENSITIVITIES

In this section we will compute the LISA TDI sensitivities to vector and scalar waves. Although the LISA sensitivities to tensor waves have already been presented in an earlier publication [31], for the sake of comparison we will include them in the sensitivity plots presented in this section. We will specialize our calculations to

the equilateral-triangle configuration: armlength 1 = armlength 2 = armlength 3 = L ($L \simeq 16.7$ light seconds) since the LISA arm lengths will differ by at most a few percent, and any corrections to our results will be to this level of accuracy [24,30,31].

The LISA sensitivity to tensor GWs has been traditionally taken to be equal to (on average over the sky and polarization states) the strength of a sinusoidal GW required to achieve a signal-to-noise ratio of 5 in a one-year integration time, as a function of Fourier frequency. Although in the case of vector waves the average over the polarization states can be performed by implementing the same procedure described in [24,31] for tensor waves, in general this cannot be done for scalar waves. This is because the two scalar fields are mutually orthogonal (and independent), as one is purely longitudinal and the other purely transverse to the direction of propagation of the wave.

In order to compute the LISA sensitivities, we will use the following expressions for the power spectra of the noises affecting the X, α, ζ, E, P, and U combinations [31] (see Fig. 2):

$$S_X(f) = [8\sin^2(4\pi fL) + 32\sin^2(2\pi fL)]S_y^{\mathrm{pm}}(f)$$
$$+ 16\sin^2(2\pi fL)S_y^{\mathrm{op}}(f) \tag{17}$$

$$S_\alpha(f) = [8\sin^2(3\pi fL) + 16\sin^2(\pi fL)]S_y^{\mathrm{pm}}(f) + 6S_y^{\mathrm{op}}(f) \tag{18}$$

$$S_\zeta(f) = 24\sin^2(\pi fL)S_y^{\mathrm{pm}}(f) + 6S_y^{\mathrm{op}}(f) \tag{19}$$

$$S_E(f) = [32\sin^2(\pi fL) + 8\sin^2(2\pi fL)]S_y^{\mathrm{pm}}(f)$$
$$+ [8\sin^2(\pi fL) + 8\sin^2(2\pi fL)]S_y^{\mathrm{op}}(f) \tag{20}$$

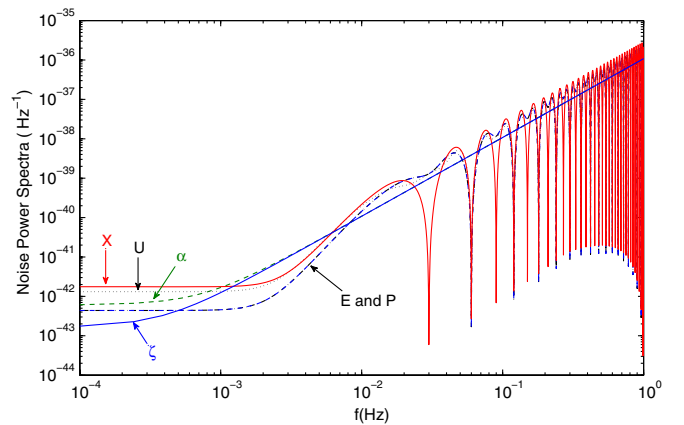FIG. 2 (color online). Noise spectra in the X, α, ζ, E, P, and U time-delay interferometric combinations accounting for the nominal proof mass and optical path noises. The varying depths of the minima in the high-frequency ranges of X, E, and P are an artifact of numerically calculating these functions at discrete frequencies.

$$S_P(f) = [8\sin^2(2\pi fL) + 32\sin^2(\pi fL)]S_y^{pm}(f)$$
$$+ [8\sin^2(2\pi fL) + 8\sin^2(\pi fL)]S_y^{op}(f) \quad (21)$$

$$S_U(f) = [16\sin^2(\pi fL) + 8\sin^2(2\pi fL)$$
$$+ 16\sin^2(3\pi fL)]S_y^{pm}(f) + [4\sin^2(\pi fL)$$
$$+ 8\sin^2(2\pi fL) + 4\sin^2(3\pi fL)]S_y^{op}(f), \quad (22)$$

where $S_y^{pm}(f) = 2.5 \times 10^{-48}[f/1\text{ Hz}]^{-2}\text{ Hz}^{-1}$ is the spectrum of the relative frequency fluctuations due to each

proof mass, and $S_y^{op}(f) = 1.8 \times 10^{-37}[f/1\text{ Hz}]^2\text{ Hz}^{-1}$ is the spectrum of optical path (mainly shot and beam pointing) noise. Both these noises can be regarded as the main limiting noise sources for LISA [7,31].

Gravitational wave sensitivity is the wave amplitude required to achieve a given signal-to-noise ratio. We calculate it in the conventional way, requiring a signal-to-noise ratio of 5 in a one year integration time: $5\sqrt{S_k(f)B}$/(root-mean-squared gravitational wave response for data combination k), where k is X, α, ζ, E, P, U, and S_k
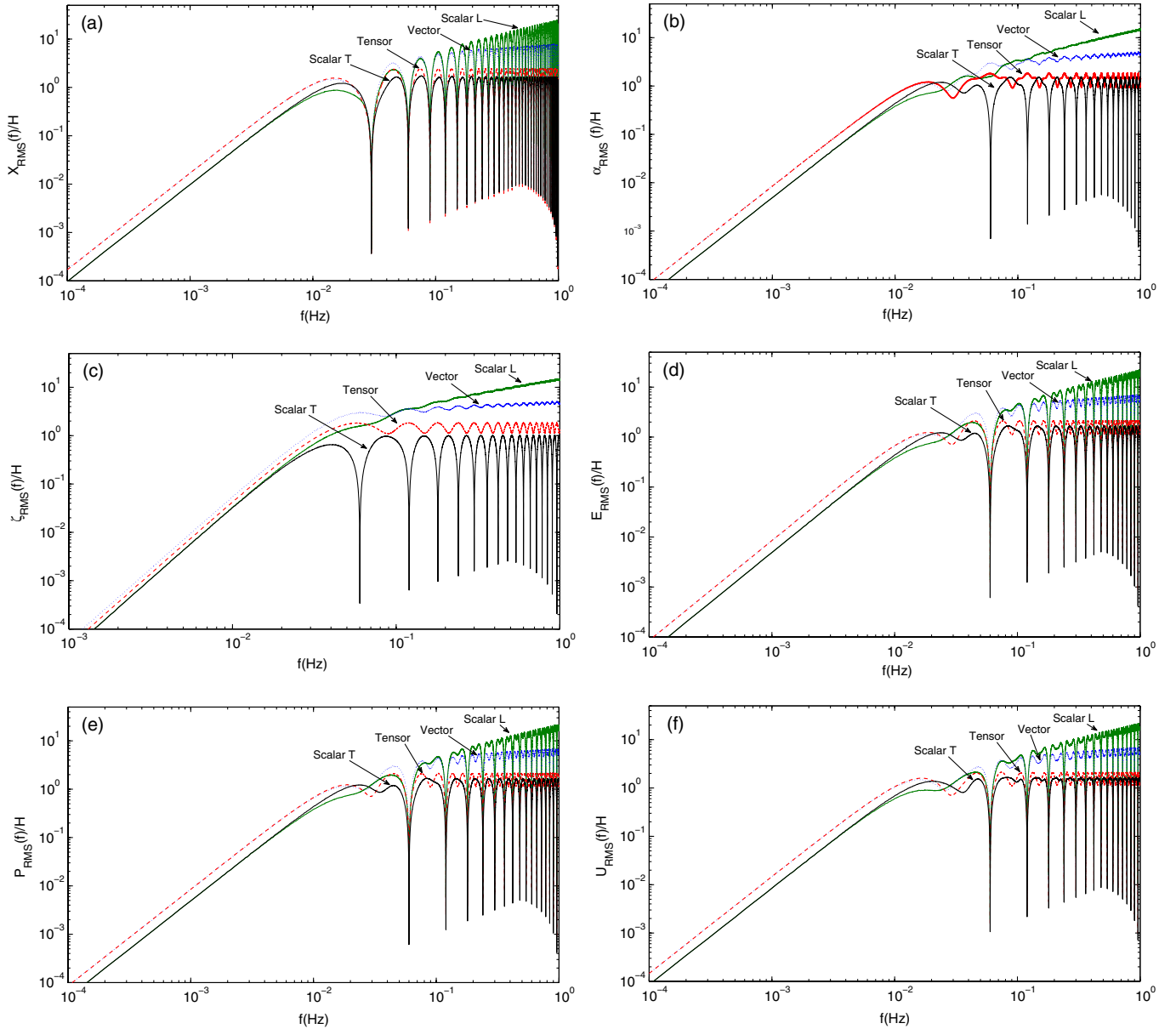

FIG. 3 (color online). Root-mean-square responses of the (a) X, (b) α, (c) ζ, (d) E, (e) P, and (f) U TDI combinations to tensor ($s = \pm 2$), vector ($s = \pm 1$), and scalar ($s = 0$) gravitational waves. The former two have been treated as elliptically polarized waves, while the two waveforms characterizing the scalar signals have not as they are mutually orthogonal (and therefore independent). The sensitivities have been calculated by assuming an ensemble of sinusoidal signals uniformly distributed on the celestial sphere and (in the case of tensor and vector radiation) randomly polarized. In the high part of the frequency band the rms values of the vector and scalar-longitudinal waves are noticeably larger than the tensor and scalar-transverse signals, and grow with the Fourier frequency.

is the total noise power spectrum for TDI combination k. The bandwidth, B, was taken to be equal to one cycle/year (i.e. 3.17×10^{-8} Hz).

We have assumed the vector waves to be elliptically polarized and monochromatic, with their wave functions, $(h^{(2)}, h^{(3)})$, written in terms of a nominal wave amplitude, H, and the two Poincaré parameters, (Φ, Γ), in the following way:

$$h^{(2)}(t) = H \sin(\Gamma) \sin(\omega t + \Phi), \tag{23}$$

$$h^{(3)}(t) = H \cos(\Gamma) \sin(\omega t). \tag{24}$$

For scalar signals instead, the two wave functions, $(h^{(1)}, h^{(6)})$, have been treated as independent and we calculated the TDI sensitivities to each of these two polarizations. For both vector and scalar signals we averaged over source direction by assuming uniform distribution of the sources over the celestial sphere; in the case of vector signals we also averaged over elliptical polarization states uniformly distributed on the Poincaré sphere for each source direction. The averaging was done via Monte Carlo integration with 4000 source position/polarization state pairs per Fourier frequency bin and 7000 Fourier bins across the LISA $(10^{-4} - 1)$ Hz band [24,31].
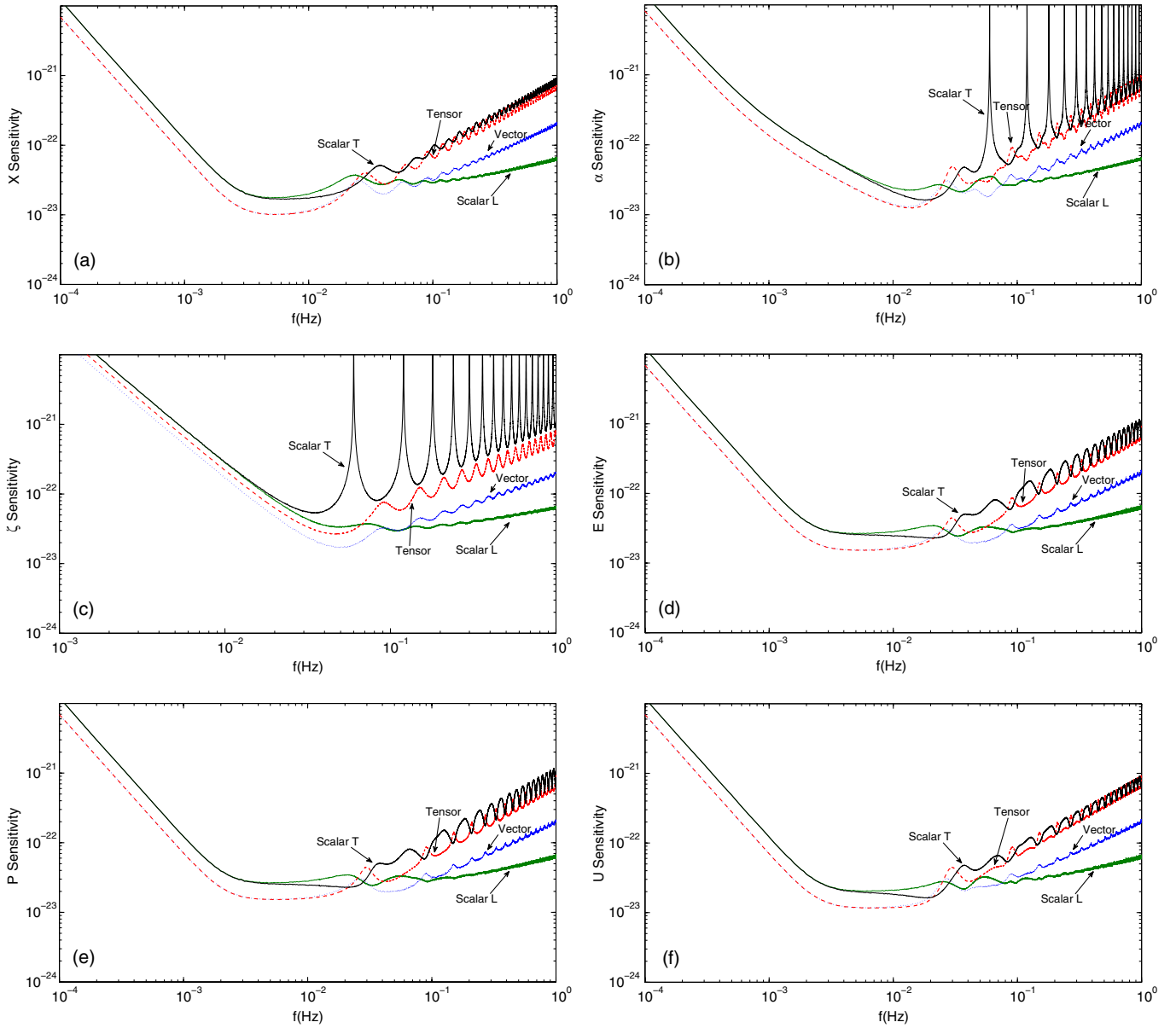

FIG. 4 (color online).   Sensitivities of the (a) X, (b) α, (c) ζ, (d) E, (e) P, and (f) U TDI combinations to gravitational waves with tensor $(s = \pm 2)$, vector $(s = \pm 1)$, and scalar $(s = 0)$ components. Consistently with Fig. 3, we may notice how more sensitive the various TDI combinations are to vector and scalar-longitudinal signals than to tensor and scalar-transverse waves in the high-frequency part of the LISA frequency band. See the text for more details.

Figure 3 shows the root-mean-squared (rms) responses of the TDI combinations (a) X (unequal-arm Michelson), (b) α (Sagnac), (c) ζ (symmetrized Sagnac), (d) E (monitor), (e) P (beacon), and (f) U (relay) to tensor (already derived in [24] and shown here for comparison), vector and scalar gravitational waves. In the high part of the LISA frequency band we may notice that the rms responses to vector and scalar-longitudinal waves are significantly larger than those to tensor and scalar-transverse signals. In particular, the scalar-longitudinal rms response grows with the frequency at a much faster rate than the others.

In order to physically understand this effect, let us compare a one-way Doppler response (that measured on board spacecraft 1, for instance) to a "pulse" tensor wave against that due to a scalar-longitudinal pulse wave. A tensor signal propagating orthogonally to the light beam (direction for which the one-way Doppler response can reach its maximum magnitude in this case) will only interact with the light for the brief instant its wave front crosses the light beam. On the other hand, if a scalar-longitudinal wave propagates along the direction between the two spacecraft (over which the Doppler response will achieve its maximum in this case), the frequency of the light will be affected by the gravitational wave for the entire time L it takes the wave to propagate from one spacecraft to the other, resulting into an amplification of the frequency shift when the wavelength of the wave is shorter than the interspacecraft distance, L. The above considerations become apparent by considering in both cases the one-way Doppler response y' in the Fourier domain. For a tensor signal, the modulus squared of the Fourier transform of Eq. (16) with $\hat{k} \cdot \hat{n} \to 0$ becomes equal to

$$|\tilde{y}'(f)_{\hat{k} \cdot \hat{n} \to 0}|^2 = \sin^2(\pi f L)|\tilde{h}(f)|^2, \qquad (25)$$

with $\tilde{h} \equiv \tilde{h}^{(4)}\cos(2\phi) + \tilde{h}^{(5)}\sin(2\phi)$. In the case of a scalar-longitudinal signal instead, it is easy to show that, in the limit of $\hat{k} \cdot \hat{n} \to -1$, the modulus squared of the Fourier transform of Eq. (16) becomes equal to

$$|\tilde{y}'(f)_{\hat{k} \cdot \hat{n} \to -1}|^2 = (\pi f L)^2|\tilde{h}^{(1)}(f)|^2. \qquad (26)$$

From the above two expressions we may conclude that, at frequencies larger than the inverse of the one-way-light-time and for tensor and scalar-longitudinal waves of comparable amplitudes, the maximum one-way Doppler response to a scalar-longitudinal signal will be larger than the corresponding maximum response to a tensor wave by roughly a factor of $\pi f L$. This example implies that the rms of the TDI responses to scalar-longitudinal signals will be larger than those to tensor waves in the high-frequency region of the LISA band. Similar considerations can be made for understanding the differences between the rms responses to tensor and vector waves.

In the low-frequency limit (for $f < 5 \times 10^{-3}$ Hz) the tensor and vector rms responses coincide, while those for the two scalars also coincide with each other but are smaller by a factor of about 2 than those for tensor and vector waves. This is because the two scalar waves, being mutually orthogonal, have been treated as independent rather than elliptically polarized like the vector and tensor waves.

In Fig. 4 we then plot the corresponding sensitivities of the TDI combinations to the tensor, vector, and the two scalar polarization components. The characteristic behavior of the rms responses to scalar-longitudinal and vector signals shown in Fig. 3 folds into the plots presented here. At high frequencies the sensitivity to scalar-longitudinal waves is significantly better than that to tensor, vector, and scalar-transverse waves. At 1 Hz, for instance, the sensitivity of scalar-longitudinal signals is about a factor of $f L \simeq 17$ better than that to tensor waves, while it is only a factor of 3 better than the sensitivity to vector signals.

Another interesting feature noticeable in Fig. 4 is the lack of sensitivity of the Sagnac combinations (b) α and (c) ζ to scalar-transverse waves at frequencies equal to integer multiples of the inverse of the one-way-light-time. We have verified this result analytically and found that indeed, at these frequencies, the Sagnac responses are identically equal to zero independently of the direction of propagation of the signal.

IV. SUMMARY AND CONCLUSIONS

The main results of our work have been that (i) LISA is more sensitive to scalar-longitudinal and vector signals than to tensor waves in the high part of its frequency band, and (ii) at low frequencies its sensitivities to tensor and vector signals are equal and somewhat better than those to scalar waves. We have also found that the LISA TDI Doppler responses to scalar-longitudinal waves propagating along any of the three LISA arms will experience an amplification proportional to the arm length. These results, together with the LISA capability for constructing three independent TDI combinations in the high part of its observational frequency band, should provide LISA with the capability for assessing the polarization of the waves it will detect. This will be the topic of our forthcoming investigation.

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