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Length of a minimum as predictor of next solar cycle's strength

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[1] Motivated by a prevailing view that a long minimum leads to a weak sunspot cycle, we estimate the correlation coefficients between the length of a cycle minimum and (i) the following cycle's peak, (ii) the preceding cycle's peak, (iii) following peak minus preceding peak and (iv) depth of minimum. Using both sunspot number and spot area data, we find that a long minimum is both followed and preceded by weak cycles. Similarly short minima are followed and preceded by strong cycles. Consistent with these results, we find no correlation between the length of a cycle minimum and the difference in peaks of the following and preceding cycles. From sunspot number data, for longer-than-average minima, five following cycle peaks were lower than that of the preceding cycles' peaks, while four were higher. Following shorter-than-average minima, seven cycle peaks were higher than the preceding peaks and seven were lower. Therefore one cannot predict from the length of a minimum whether the next cycle will be stronger or weaker than the preceding cycle. Thus we cannot predict whether cycle 24 will be stronger or weaker than 23. We also find that there is a strong anticorrelation between the length of a solar cycle minimum and the depth of that minimum. We define the depth as the least spot number or spot area (13-rotation averaged) within the span of a cycle minimum. We speculate that this anticorrelation is due to the longer time available for annihilation of late cycle toroidal flux across the equator in the case of a longer minimum.
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1. Introduction and Motivation

[2] Recently *Dikpati et al.* [2006] made a prediction of the future solar cycle (cycle 24) using a dynamo model calibrated to the Sun. Previously the most widely used technique for solar cycle predictions was based primarily on statistical and empirical calculations following the characteristics of previous cycles' polar field amplitudes. *Dikpati et al.* [2006] predicted that cycle 24 would have a peak 30-50% higher than cycle 23. This was in contrast to a number of other predictions obtained using the polar field amplitude of cycle 23 that cycle 24 would be smaller than cycle 23. *Kane* [2007] and *Pesnell* [2008] give detailed discussions of all published predictions of cycle 24.

[3] This high cycle prediction had a significant influence on the deliberations of the Solar Cycle 24 Prediction Panel convened by NOAA and sponsored by NASA, whose first meeting was in October 2006. The procedure of the panel is for each member to vote on whether the next cycle will be high, medium or low, based on each member's assessment of the value of each of the forecasts previously reported in the literature or to the panel. At the first meeting where a vote was taken, about 2/3 of the panel members favored a high cycle. In May 2007, by contrast, 1/2 of the members thought the cycle would be low, and by the fall of 2008, 3/4ths of the members thought cycle 24 would be low.

[4] What did the Sun do during this time that may have changed the perspectives of some panel members? The minimum period at the end of cycle 23 became longer and longer, with no clear sign until 2009 of the start of cycle 24. The previous six minima were relatively short, between 10 and 20 months (average of 16 months). The minimum at the end of cycle 23 is now more than twice as long as that average.

[5] The length of a cycle and next cycle's peak has been shown to be well anticorrelated [*Kane*, 2008], but to our knowledge no one has studied the relationship between the lengths of minima and cycle peaks. Rather, implicitly or explicitly, some of us may have a prevailing thought that the longer the minimum period, the more likely the next cycle will be a weak one, or at least weaker than the previous one. In this paper we investigate whether there is historical evidence to support these inferences. We compute, using 13-rotation averaged sunspot area data, previously used by *Dikpati et al.* [2006] and monthly smoothed sunspot number data downloaded from the SIDC (Solar Influences Data Analysis Center; www.sidc.be) website, the correlations between the lengths of minima and cycle peaks of preceding and following cycles. We specifically address the following questions: (i) Are the length (suitably defined) of minima and the following cycles' peaks strongly anti-correlated? (ii) Is there any such anti-correlation between the length of minima and preceding cycles' peaks? (iii) Can the length of a minimum be used to predict whether the next cycle's peak is higher or lower than the previous cycle's?

2. Calculations and Results

[6] To calculate the correlations we want, we read directly from the sunspot number and area tables the peak of each cycle, and the minimum value between cycles. There is no unique way to estimate the "length" of minima. We chose to pick a cutoff value of sunspot number and area below which the sun is defined to be in minimum phase. These values must be high enough that the minimum value between cycles is significantly below the cutoff value, but low enough to limit the minimum phase to a modest fraction of the length of the weakest cycles.

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Table 1. Length and Depth of Cycle Minima, and Peaks of Preceding and Following Cycles, for Spot Number Data

Minimum Between Cycles	Length of Minima (months)	Depth of Minima	Preceding Peak	Following Peak
0–1	32	8.4	92.6	86.5
1–2	8	11.2	86.5	115.8
2–3	18	7.2	115.8	158.5
3–4	14	9.5	158.5	141.2
4–5	47	3.2	141.2	49.2
5–6	79	0.0	49.2	48.7
6–7	58	0.1	48.7	71.5
7–8	22	7.3	71.5	146.9
8–9	15	10.6	146.9	131.9
9–10	27	3.2	131.9	98.0
10–11	17	5.2	98.0	140.3
11–12	49	2.2	140.3	74.6
12–13	49	5.0	74.6	87.9
13–14	46	2.7	87.9	64.2
14–15	48	1.5	64.2	105.4
15–16	24	5.6	105.4	78.1
16–17	38	3.5	78.1	119.2
17–18	16	7.7	119.2	151.8
18–19	19	3.4	151.8	201.3
19–20	15	9.6	201.3	110.6
20–21	18	12.2	110.6	164.5
21–22	11	12.3	164.5	158.5
22–23	19	8.3	158.5	120.8

[7] We tested various values, and settled on 15 for sunspot number and 180 (in units of 10^{-6} of visible hemisphere) for spot area. We have also estimated the length of minima using other neighboring values, such as 10, 20, 25 for spot number. Using an algorithm from Numerical Recipes [*Press et al.*, 1992] we computed the correlation among these minima-lengths, and found that these lengths correlate strongly ($r = 0.98 - 0.99$) with one another. This means that the *relative* lengths of minima all increase (decrease) by the same percentage when the threshold is increased (decreased). This implies that other correlations found using the length of minima should not be sensitive to the threshold chosen, and we have found that to be the case.

[8] The sunspot number and area data taken from the full data sets are shown in Tables 1 (for spot number) and 2 (for spot area). The answers to the questions we posed at the end of §1 are contained in Table 3.

[9] We can see from the first 3 rows of Table 3 that there is a significant anti-correlation between the length of a minimum and the peak of the following cycle, supporting the prevailing conception among us. One might be tempted from this to predict a low cycle 24 based upon this strong negative correlation between the minima-length and the next cycle's strength. However, this might be risky, because we find that there is also a somewhat smaller, but still significant, anti-correlation between the length of the minimum and the preceding cycle peak (rows 4–6 in Table 3). Taken together, these correlations could be due to longer term trends in solar cycle peak data for the 19th and 20th centuries, as noted by many others [e.g., *Kane*, 2002].

[10] Most of the prediction methods based on statistical approaches consider the previous cycle's properties (see detailed discussions by *Hathaway et al.* [1999]). Given the above result, that the length of the minima anti-correlate significantly with the following cycles' peaks as well as with the preceding cycles' peaks, we examined whether we can say something about the following n*th* cycle's amplitude relative to the preceding (n − 1)*th* one's, based on the length of the minimum between the n*th* and (n − 1)*th* cycles. In

Table 2. Length and Depth of Cycle Minima, and Peaks of Preceding and Following Cycles, for Spot Area Data

Minimum Between Cycles	Length of Minima (months)	Depth of Minima	Preceding Peak	Following Peak
11–12	50.7	21.58	——	1289.12
12–13	44.5	73.13	1289.12	1492.72
13–14	46.5	36.59	1492.72	1066.35
14–15	42.9	16.47	1066.35	1468.71
15–16	18.5	78.52	1468.71	1367.88
16–17	30.4	68.81	1367.88	2068.37
17–18	9.9	124.39	2068.37	2573.93
18–19	17.5	55.25	2573.93	3368.42
19–20	20.0	67.82	3368.42	1590.58
20–21	10.5	166.02	1590.58	2446.60
21–22	17.2	120.13	2446.60	2434.86
22–23	23.7	80.09	2434.86	1916.77

Table 3. Correlations Between Length of Cycle Minima and Cycle Peaks, With Probabilities They Could Occur by Chance[a]

Data Type	Correlation	Significance
Length of Minimum With Following Cycle Peak		
spot number	−0.75	3×10^{-5}
total spot area	−0.71	1×10^{-2}
spot area in N&S separately	−0.67	3×10^{-4}
Length of Minimum With Preceding Cycle Peak		
spot number	−0.59	3×10^{-3}
total spot area	−0.54	9×10^{-2}
spot area in N&S separately	−0.47	3×10^{-2}
Length of Minimum With Difference Between Following and Preceding Cycle		
spot number	−0.14	5×10^{-1}
total spot area	−0.10	8×10^{-1}
spot area in N&S separately	−0.13	6×10^{-1}

[a]Column labeled "significance."

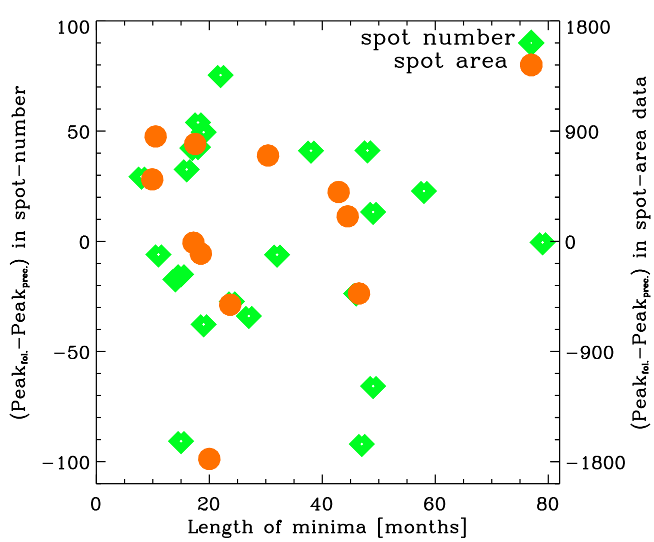

Figure 1. Scatter-plot for difference between following and preceding cycles' peaks as function of length of minima between following and preceding cycles.

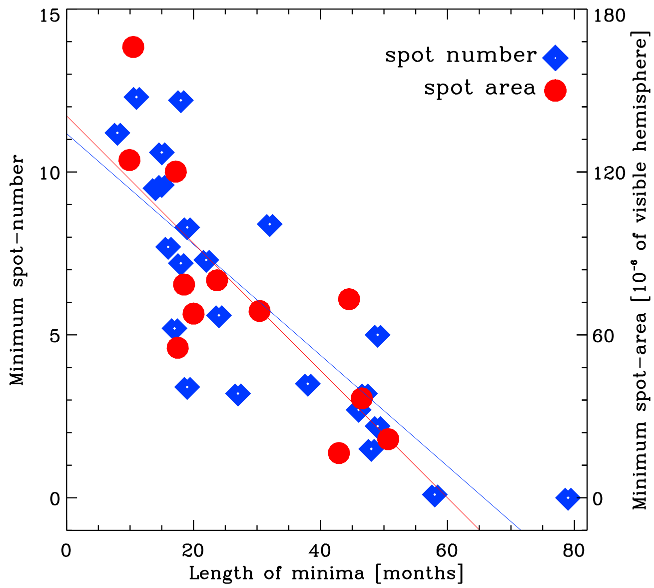

Figure 2. Scatter-plot for length and depth of minima.

order to do that, we correlate the length of minimum with the difference in cycle peak between the preceding and following cycles (rows 7–9 in Table 3). By taking the difference between following and preceding cycle-peaks, we eliminate all longer term trends.

[11] We find there is no significant correlation. The correlation $r = -0.13$ for spot number data, and $r = -0.10$ for area data. There is about a 70% chance that these correlations are random. Figure 1 presents in green diamonds a scatterplot of the differences between the following cycle's spot number and the preceding cycle's spot number as function of minima-lengths. The same for area data is shown in red circles. Breaking up the spot area data into northern and southern hemispheres does not improve the significance. In terms of individual minima, from the spot number data we find that in five cases a longer than average minimum is followed by a lower cycle, while in four cases a longer minimum is followed by a higher cycle. Seven short minima are followed by higher cycle, but seven are also followed by a lower cycle.

[12] We also examined whether there is any correlation between the length of a minimum and its depth. We define depth as the least spot number or the least spot area within the span of a cycle minimum. In terms of correlation coefficient r (correlating column 2 with column 3 in Tables 1 and 2) we get $r = -0.75$ for spot number, and $r = -0.79$ for spot area. The probabilities that either of these values is due to chance is 1–2 $\times$ 10^{-3}. Figure 2 shows a scatterplot with linear fits to both data sets. The negative correlation is seen clearly. We give a possible physical explanation for this result below.

[13] What might the strong negative correlation between length of a minimum and the depth of that minimum be telling us about the Sun? One obvious explanation is simply that a delay in the start of a new cycle leads to a longer period during which sunspot number and area remain below their cutoff values. But that may not be the whole story. Low latitude spots late in a cycle imply the presence of toroidal fields below the surface in low latitudes in both hemispheres (see Figure 3). Their close proximity for a more extended time may lead to more cancellation of toroidal flux between the two bands, by turbulent diffusion and/or instabilities [*Cally et al.*, 2003], leading to fewer spots seen at the surface in each hemisphere. Such cancellation occurs in all dynamo models that include shearing as a mechanism for generating spot-producing toroidal fields [*Wang and Sheeley*, 1991; *Dikpati and Charbonneau*, 1999; *Markiel and Thomas*, 1999] no matter whether meridional circulation is included in the model or not.

3. Conclusions

[14] Supporting the persisting idea, we found that there is a substantial anticorrelation between the length of cycle minimum and the following cycle's amplitude. But we also found that the length of cycle minimum anticorrelates almost as well with the preceding cycle's amplitude. Taken together these anticorrelations suggest that there are longer term trends in the data that limit the ability to predict whether the amplitude of the next cycle will be higher or lower than that of the previous cycle, when the predictor used is the length of the previous minimum.

[15] One way to eliminate any longer term trends, regardless of origin, is to correlate the length of minimum with the difference in amplitude between the following and preceding cycles. We have shown that there is no such correlation. This means that it is not possible to predict whether the next cycle will be stronger or weaker than the previous one when we use the length of minimum as the predictor. Therefore the current long minimum at the end of cycle 23 does not necessarily portend a weaker cycle 24 than 23. The rise of cycle 24 is underway and we all must wait to see the peak of cycle 24 to see which prediction of the peak is most accurate.

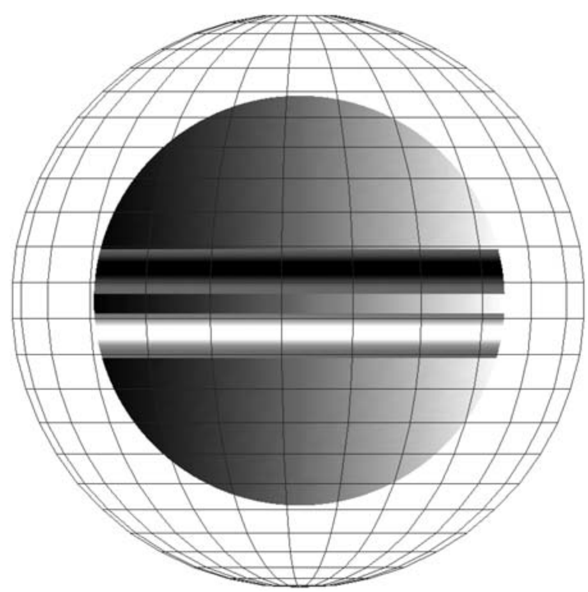

Figure 3. A schematic diagram showing a configuration of solar minimum toroidal bands as would be generated at the base of the convection zone by the shearing mechanism of a dynamo.

Given the long minimum we are currently experiencing, the peak of cycle 24 may not occur until 2013 or even 2014.

[16] Curiously we found a strong anticorrelation between the length and depth of a minimum. This can be explained from dynamo processes oppositely directed toroidal bands in close proximity on the two sides of the equator have more time in the case of a longer minimum to annihilate each other, leading to fewer or no eruptions of low-latitude spots.

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