Computational Performance of Carsharing Fleet-Sizing Optimization

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Abstract. *Amid the expansion of shared-economy products, carsharing services aim to offer short-term car rentals. An optimized fleet-size allocation for each service station is important for serving as many clients as possible, maximizing the company's profit. This work proposes and compares the computational performance of two Mixed-Integer Linear Programming formulations to support the carsharing simulation. The performed simulations varied the number of offered vehicles, and the number of clients looking for the service. Real spatial data from the city of São Paulo, Brazil, were used on the simulations. Results show that the formulation which does not use the Big-M method finds the global optimal solution faster and can scale up better.*

1. Introduction

Transportation plays an important role in the society by enabling people to commute to school, work, shopping and leisure activities in their cities. Improving the access to mobility was the subject of recent work, aiming to identify transport related social exclusion (Logiodice et al., 2015), suggesting new locations for pick-up and drop-off for public transportation (Monteiro et al., 2017), and building applications to integrate, visualize, analyze data of public transportation (Alic et al., 2018).

Along with inaccessibility, issues such as discomfort, low diversity of operating lines, low supply of buses at certain times, high transportation fares and extensive trip length can motivate passengers to use alternative transportation means (Monteiro et al., 2019), among which are carsharing services. Carsharing consists in offering vehicles in a "as-needed" basis. Clients can rent cars for periods as little as some minutes, avoiding the costs of owning a vehicle or renting it for a whole day (Machado et al., 2018).

In summary, there are three modalities of carsharing: round-trip, one-way and free-floating. On the round-trip, the vehicle must be returned to the same station where the rental has started. On the one-way, the vehicle can be returned to a different station. And on the free-floating, there are no stations and the vehicles can be parked on the streets (Machado et al., 2018). In all those carsharing modalities, and mainly on the station-based ones, the number of available vehicles must be determined in order to avoid unnecessary costs and to offer attractive prices (Boyacı et al., 2017; Lage et al., 2019).

Recent works use Simulation-Based Optimization (SBO) to simulate carsharing dynamics, and to support the decision-making process (Monteiro et al., 2019). This work proposes two Mixed-Integer Linear Programming (MILP) formulations to maximize carsharing profits. Our objective is to compare the computational performance of these formulations, in order to support broader and more complex analyses in the future. Real spatial data from the city of São Paulo, Brazil, were used. An analysis of the optimal solutions found by varying the number of simulated clients and the maximum number of vehicles is also shown. Results present the benefits of applying a SBO for carsharing planning and indicate that the formulation without the Big-M method runs faster and scales up better for optimizing the carsharing fleet-sizing problem.

This paper is organized in four sections. Next section discusses related work. Section 3 explains the proposed formulations. Section 4 shows and discusses the results, and Section 5 concludes this paper.

2. Related Work

SBO approaches enable the decision-maker to evaluate the impact of parameter changes, being useful to support "what-if" scenario analyses (Oliveira et al., 2015; Monteiro et al., 2019). Most of the optimization problems for carsharing are deterministic and exact, based on MILP models, and are Mono-Objective. However, some works did not follow this pattern, and chose to solve Multi-Objective problems, for example.

Correia and Antunes (2012) proposed a MILP model to maximize carsharing profits considering all the revenues and costs involved. The work optimizes the locations for carsharing stations, balancing the fleet of vehicles among the stations on the one-way modality. The authors evaluated the proposed MILP model for a case study in Lisbon, Portugal, showing the impact of the stations' location for different behaviour of clients.

Jorge et al. (2014) compared two methods for fixing the unbalancing of vehicles among stations, in Lisbon, Portugal. That unbalancing happens due to different demands of clients on the one-way modality. On the one-way, even if the number of vehicles distributed through the stations at the day's beginning is suitable, demand peaks can quickly occupy all vehicles from one station. In that case, other clients from that same station will not be served, even if there are idle vehicles in other stations. The model based on mathematical formulations achieved solutions with better profits, mainly while considering the costs of relocating the vehicles.

Many carsharing companies do not offer one-way modality, since the costs of relocating vehicles can raise carsharing prices, making the rentals unattractive. Jorge et al. (2015) proposed a MILP model to optimize round-trip carsharing to also offer one-way rentals in Boston, USA. As expected, results showed that including one-way services in a optimized way could increase the number of clients served.

Lu et al. (2017) proposed a stochastic MILP based on Benders decomposition. The optimization was applied on data from the Boston-Cambridge area in Massachusetts, USA, and analyzed the percentage of the fleet used, the number of vehicles, relocation costs, and QoS (Quality of Service). The results indicated that if the client demands are generated by pricing and strategic customer behavior instead of by natural market penetration and user adoption, the one-way profit can decrease in comparison with round-trip profits.

A Multi-Objective MILP formulation is proposed by Boyacı et al. (2015) to maximize user benefit and carsharing net revenue using electric vehicles. Their work was extended in Boyacı et al. (2017) by proposing a procedure to cluster the stations in order to reduce the number of variables, and to consider the relocation problem when the carsharing service allows reservations.

Bruglieri et al. (2018) proposed a Multi-Objective MILP formulation for the Electric Vehicle Relocation Problem for one-way carsharing in Milan, Italy. In order to maximize the profits, the authors' formulation has three objectives: minimizing the number of workers needed to relocate vehicles; maximizing the number of relocations; and minimizing the lengthiest relocation route performed. The computational performance and the optimal values between approximate optimization methods and exact ones were compared. Results show the benefits of using the approximate method instead of a slower optimization method.

Monteiro et al. (2019) proposed a MILP formulation for round-trip and one-way fleet-sizing. The formulation objective was to maximize the predicted profit. The authors evaluated different scenarios from the city of São Paulo, Brazil, varying the number of clients, driving distance, rental duration, two models of cost calculation and two models of rental prices. The results showed that round-trip carsharing can overcome the profit from the one-way mode in scenarios with higher rental durations.

This paper differs from the related work by proposing and comparing the performance of two MILP formulations for fleet-sizing optimization of carsharing. The proposed formulations are based on round-trip, and experimental results were applied on real spatial data from the city of São Paulo. The results can be useful for carsharing companies, conventional car-rental services, and other shared-mobility services such as bikesharing, supporting the resources allocation. The SBO methods are described in the next section.

3. Simulation-Based Optimization

This section presents the simulation performed and the proposed formulations. Both formulations have the same load of data, and therefore, generate the same global optimal results. Figure 1 presents the location of 100 stations, randomly generated for the experiments. All the generated locations are placed in a street in São Paulo. Therefore, regions with larger total street length are more likely to receive a carsharing station. That procedure also avoids locating stations on regions with only water, woods or no driving access.

As detailed by Monteiro et al. (2019), the number of generated clients varies according to the population in the district where the station is located. A São Paulo district is the smallest official spatial unit adopted by the local government. Thus, the demand is divided throughout the city, simulating more clients in regions with larger population. Rental start and end times and the corresponding driving distance are generated randomly, and both follow an uniform distribution. The set of stations and clients are only used as input for the proposed formulations. Different data can be applied to simulate broader case studies.

Subsection 3.1 describes the formulation based on the Big-M method. Subsection 3.2 describes the formulation without the Big-M method. The Big-M method consists in defining big enough constants and multiply them to specific variables on the objective function or constraints in order to assure the feasibility of some solutions (Bazaraa et al., 2011). The formulation presented in the following subsection uses the Big-M method to guarantee that earlier clients arriving at the stations will have priority on being served.

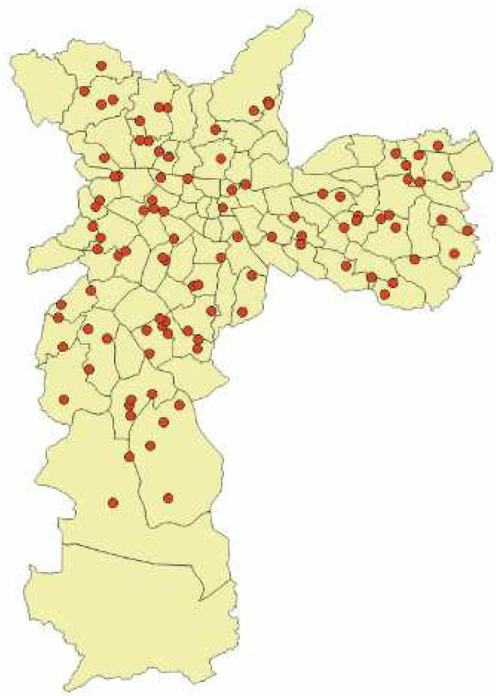

Figure 1. Generated locations for the carsharing stations

3.1. Formulation with Big-M

Table 1 presents the variables used and Table 2 presents the constants used in this formulation. Equation 2 presents the objective function, with the goal to maximize the difference between total revenue and total cost, generating the profits.

Revenue and costs are calculated using models presented by Monteiro et al. (2019). The revenue from each rental was fared as R$10[1] per hour plus R$0.90 per driven kilometer, with a minimum fare of R$20. Equation 1 defines the revenue R_{x_s}. The cost C_{x_s} is calculated as R$0.50 per driven kilometer, and cost C_s is defined as R$13 per day and per vehicle, indicating the vehicle's depreciation along the time of use.

$$R_{x_s} = max(20, (T_{x_s}^{end} - T_{x_s}^{start}) \times 10 + D_{x_s} \times 0.50) \quad (1)$$

Table 1. Model Variables

Variable	Description
$s \in \mathbb{S}$	Carsharing station
n_s	Number of vehicles to be allocated in station s
$x_s \in \mathbb{X}$	Client willing to rent a vehicle from station s

[1]Brazilian currency: Reais (R$). For comparison, the exchange rate on August 13, 2019, was of R$3.96 per US dollar

Table 2. Model Constants

Constant	Description
P_s	Number of parking slots in station s
C_s	Cost of maintaining a vehicle
$C_{x_s^i}$	Cost made by x_s for using the vehicle
R_{x_s}	Revenue obtained for serving client x_s
D_{xs}	Distance driven by x_s
$T_{x_s}^{start}$	Rental start time for client x_s
$T_{x_s}^{end}$	Rental end time for client xs
M_s	(Big-M) Maximum number of clients that station s can serve

$$max \sum_{x_s \in \mathbb{X}} R_{x_s} x_s - \sum_{x_s \in \mathbb{X}} C_{x_s} x_s - \sum_{s \in \$} C_s n_s \tag{2}$$

Subject to:

$$x_s \leq n_s + \sum_{u_t \in \mathbb{X}: T_{u_t}^{end} < T_{x_s}^{start}} u_t - \sum_{e_s \in \mathbb{X}: T_{e_s}^{end} < T_{x_s}^{start}} e_s \qquad \forall x_s \in \mathbb{X} \tag{3}$$

$$M_s x_s \geq n_s + \sum_{u_t \in \mathbb{X}: T_{u_t}^{end} < T_{x_s}^{start}} u_t - \sum_{e_s \in \mathbb{X}: T_{e_s}^{end} < T_{x_s}^{start}} e_s \qquad \forall x_s \in \mathbb{X} \tag{4}$$

$$n_s \leq P_s \qquad \forall s \in \$ \tag{5}$$

$$\mathbb{X} \in \{0, 1\} \tag{6}$$

$$n_s \in \mathbb{N}^0 \tag{7}$$

Inequation 3 limits client x_s to only be served if there is at least one available vehicle. Inequation 4 ensures that client x_s will be served if there is at least one available vehicle. Inequation 5 limits the number of vehicles to be allocated in station s to the number of existent parking spots P_s. Inequation 6 defines the client's variables as binaries. Finally, constraint 7 defines variables n_s as positive integers including zero.

The Big-M method applied on Inequation 4 is important to balance both sides of that inequation. If the Big-M was not used, variable x_s, whose value is at most equal to one, would also limit the inequation's right hand side to one. The Big-M multiplying the x_s makes the left hand side have a value greater than one when variable x_s is equal to one, and makes the left hand side equal to zero when variable x_s is zero.

Although this formulation is relatively short and simple, constraint 4 can reduce the computational performance of the formulation. Next section presents an alternative version of this formulation avoiding the use of the Big-M method.

3.2. Formulation without Big-M

Avoiding to use the Big-M implies the need of creating additional variables and constraints. Table 3 presents the variables used and Table 4 presents the constants used in this formulation.

Table 3. Model Variables

Variable	Description
$s \in \mathbb{S}$	Carsharing station
$v_s^i \in \mathbb{V}$	i^{th} vehicle that can be allocated on the station s
$x_s^i \in \mathbb{X}$	Client sorted by rental start (order indexed by i)

Table 4. Model Constants

Constant	Description
C_s	Cost of maintaining a vehicle
$C_{x_s^i}$	Cost made by x_s^i for using the vehicle
$R_{x_s^i}$	Revenue obtained for serving client x_s^i
$D_{x_s^i}$	Distance driven by x_s^i
$T_{x_s^i}^{start}$	Rental start time for client x_s^i
$T_{x_s^i}^{end}$	Rental end time for client x_s^i

The first change in the variables consists in splitting the number of allocated vehicles in each station n_s into several binary variables v_s^i, one for each possible vehicle. Therefore, one v_s^i is defined for each parking slot at station s. Consequently, this formulation represents the number of allocated vehicles in the station s by defining a number of P_s binary variables. This change allows constraints relating the vehicle variables directly to the client variables, since now both are binaries. Those constraints are shown in Inequation 9.

The second change consists in preprocessing the set of clients $\mathbb{X}$ in order to select a subset $\mathbb{F} \subseteq \mathbb{X}$ with only the generated clients that have a chance to be served. That preprocessing consists in leaving out of $\mathbb{F}$ all clients that, given the flow of other clients, cannot be served even if their stations have enough available vehicles for all their parking slots. Therefore, the optimization avoids using Inequation 12 to "force" unfeasible clients to be served. Since that preprocessing only implies on the case with enough available vehicles, and the simulations were restricted for round-trip, building the set $\mathbb{F}$ is a fast procedure with linear time complexity in the number of clients $O(|\mathbb{X}|)$.

Equation 8 presents the objective function, whose rationale was kept the same as in Equation 2. Constraint 9 ensures that the first clients on station s will be served by the allocated vehicles in that station. Constraint 10 limits client x_s to be served only if there is at least one vehicle available. Constraint 11 ensures beforehand that all clients that have no chance to be served will not be served. Constraint 12 guarantees that the vehicle returned by some client will be used to serve the next client from the same station. Constraint 13 defines the client variables as binaries. Finally, constraint 14 defines the vehicle variables as binaries.

$$max \sum_{x_s^i \in \mathbb{F}} R_{x_s^i} x_s^i - \sum_{x_s^i \in \mathbb{F}} C_{x_s^i} x_s^i - \sum_{s \in \mathbb{S}} C_s \sum_{v_s^j \in \mathbb{V}} v_s^j \quad (8)$$

Subject to:

$$v_s^i \leq x_s^i \qquad \forall v_s^i \in \mathbb{V} \quad (9)$$

$$x_s^i \leq \sum_{v_s^j \in \mathbb{V}: j \leq i} v_s^j + \sum_{u_t^k \in \mathbb{X}: T_{u_t^k}^{end} < T_{x_s^i}^{start}} u_t^k - \sum_{e_s^l \in \mathbb{X}: T_{e_s^l}^{end} < T_{x_s^i}^{start}} e_s^l \qquad \forall x_s^i \in \mathbb{F} \quad (10)$$

$$\sum_{x_s^i \notin \mathbb{F}} x_s^i = 0 \quad (11)$$

$$\sum_{u_s^k \in \mathbb{f}: T_{x_s^{i-1}}^{start} < T_{u_s^k}^{end} \leq T_{x_s^i}^{start}} u_s^k \leq x_s^i + \sum_{e_s^l \in \mathbb{F}: l-i \leq |\mathbb{f}|} e_s^l \qquad \forall x_s^i \in \mathbb{F}, \forall \mathbb{f} \subset \mathbb{F} \quad (12)$$

$$\mathbb{F} \subseteq \mathbb{X} \in \{0, 1\} \quad (13)$$

$$\mathbb{V} \in \{0, 1\} \quad (14)$$

The simulations were performed on a Mac mini Server (Late 2012) with S.O. macOS Mojave 10.14.6, processor Intel Core i7 2.3 GHz, and RAM of 16 GB. The models were implemented using Python 3.7, with the wrapper PuLP[2] version 1.6.0 and the solver CBC[3] version 2.10.0. Both formulations were experimentally run using the previously described data for the city of São Paulo. Next section presents the experimental results.

4. Results

This section presents the experimental results of run time and number of served clients, number of vehicles needed and profits that a carsharing company would earn. The evaluated scenarios have 1,000, 2,000, 4,000, 8,000, 16,000, 32,000 and 64,000 clients. The maximum number of vehicles and parking slots simulated were 1,000 and 5,000.

Figure 2 presents boxplots of the optimization run times for all scenarios. A maximum time limit of 30 minutes per run was set. Each boxplot represents 40 runs for each evaluated scenario. The axis "Time (seconds)" is shown in logarithmic scale to make the visual comparison easier. Boxplots in red were simulated using the proposed formulation with Big-M and with at maximum 1,000 vehicles. Boxplots in blue and green use the proposed formulation without the Big-M method; blue shows results for a fleet of 1,000 vehicles, and green corresponds to 5,000 vehicles available.

[2] https://pythonhosted.org/PuLP/
[3] https://projects.coin-or.org/Cbc

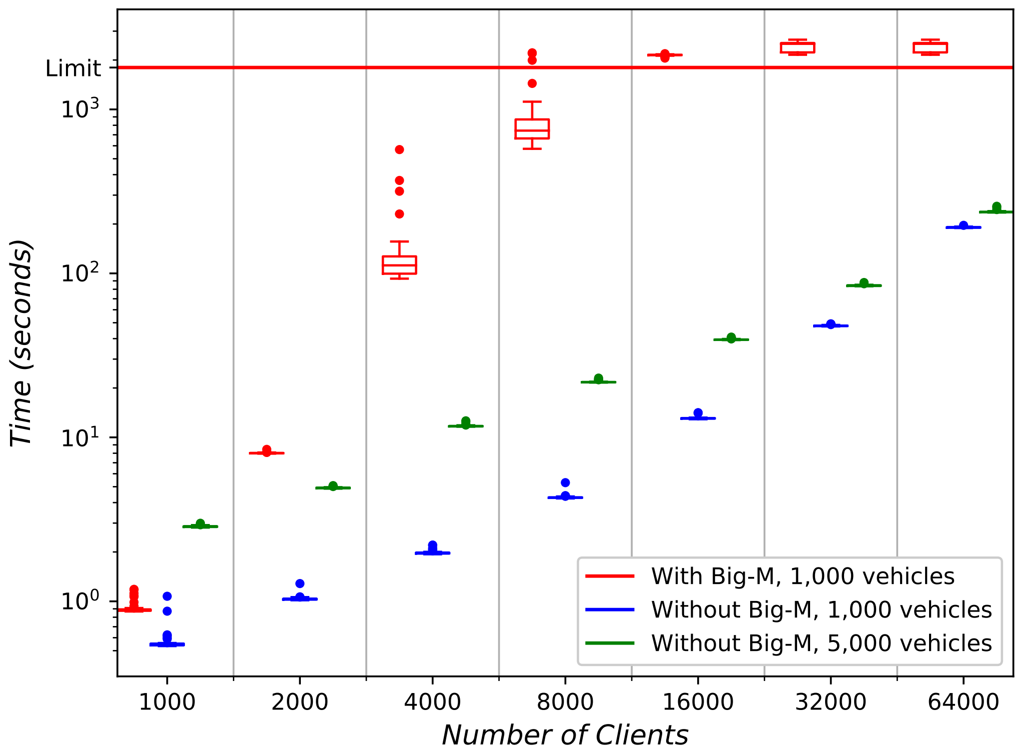

Figure 2. Time spent by the evaluated formulations

Even in the logarithmic scale, the boxes representing 50% of the data (between the first quartile, Q_1, and the third quartile, Q_3) can not be seen in Figure 2 for the blue and green boxplots. However, the red boxplots (regarding the formulations with the Big-M method) usually showed that variation more clearly. That pattern indicates that run times vary more widely in the formulation with Big-M. That higher variation can be veerified in Tables 5 and 6. Besides, the simulations with Big-M and 8,000 clients exceeded the time limit of 30 minutes in some runs. All scenarios with Big-M and more than 8,000 clients also exceeded that time limit. In those cases, the solution obtained is not guaranteed to be the optimal.

All the scenarios using the formulation without Big-M (blue and green boxplots) were solved with optimality guarantee. None of the boxplots evaluated overlap. Therefore, there is statistically significant difference between the run time of all the scenarios evaluated (Krzywinski and Altman, 2014). Thus, it can be asserted that the formulation without Big-M achieves faster run times than the formulation with Big-M. Besides, starting from 2,000 clients, the formulation without Big-M but with 5,000 vehicles is even faster than the formulation with Big-M but only with 1,000 vehicles.

Tables 5 and 6 present the basic statistics for the simulations. In both tables, the symbol M_s indicates results regarding the formulation with Big-M, and the symbol $\mathbb{F}$ indicates results from the formulation without Big-M. As shown by Table 5, the standard deviation for the scenarios with 4,000 and 8,000 clients raised quickly, when compared

Table 5. Time Spent by Running the Optimization for Low Demand (seconds)

	Number of Clients							
	1,000		2,000		4,000		8,000	
Measures	M_s	$\mathbb{F}$	M_s	$\mathbb{F}$	M_s	$\mathbb{F}$	M_s	$\mathbb{F}$
Minimum	0.87	0.53	7.95	1.02	92.84	1.94	574.96	4.24
Q_1	0.88	0.54	7.99	1.02	99.60	1.96	665.10	4.27
Median	0.88	0.54	8.00	1.03	111.90	1.97	742.23	4.29
Mean	0.91	0.57	8.02	1.04	138.45	1.99	905.03	4.32
Q_3	0.89	0.55	8.03	1.04	126.90	1.98	868.45	4.31
Maximum	1.18	1.07	8.45	1.28	568.70	2.20	2212.52	5.29
Standard Deviation	0.08	0.10	0.08	0.04	88.40	0.06	446.04	0.16

Table 6. Time Spent by Running the Optimization for High Demand (seconds)

	Number of Clients					
	16,000		32,000		64,000	
Measures	M_s	$\mathbb{F}$	M_s	$\mathbb{F}$	M_s	$\mathbb{F}$
Minimum	2055.03	12.90	2153.38	47.48	2124.38	188.99
Q_1	2141.20	13.00	2226.37	47.71	2207.79	189.74
Median	2146.41	13.05	2511.32	47.79	2519.39	190.40
Mean	2144.21	13.12	2433.02	47.93	2489.02	190.76
Q_3	2152.84	13.10	2538.97	48.03	2599.07	191.55
Maximum	2186.84	14.14	2661.79	49.22	2735.89	196.31
Standard Deviation	22.71	0.29	150.40	0.43	189.35	1.40

to the standard deviation from other scenarios. That difference was strongly reduced in Table 6, probably due to the time limit imposed.

Table 7 compares the run times of the scenario with 5,000 vehicles (green boxplots), to the run times from the scenario with 1,000 vehicles and also without the use of Big-M (blue boxplots). Since 5,000 vehicles is 5 times 1,000 vehicles, it was expected that the rate of run time would be about 5 times longer. That proportional response can be observed up to the scenario with 8,000 clients. After that, the optimization with up to 5,000 vehicles started not to be so much slower than the optimization with up to 1,000 vehicles. One hypothesis for that pattern is that the constraint presented by Equation 11 make those scenarios faster by not even letting the unfeasible clients ($x_s^i \notin \mathbb{F}$) be considered to be served along the optimization.

Tables 8 and 9 compare the optimal solutions found. The numbers of served clients, earned profits and used vehicles used tended to increase together in similar rates through the scenarios. However, that increase seemed to saturate in the scenarios with high demand of clients. Up to scenario with 8,000 clients, as the demand doubled, the percentage of increase more than doubled. But starting from demand of 16,000 clients, as the demand doubles, the percentage of increase did not change significantly. That saturation indicates that is needed more than 5,000 vehicles and parking slots for significantly

Table 7. Time Comparison Varying to 5,000 Vehicles (seconds and proportion)

	Number of Clients													
	1,000		2,000		4,000		8,000		16,000		32,000		64,000	
Stats.	Time	Rate	Time	Rate	Time	Rate	Time	Rate	Time	Rate	Time	Rate	Time	Rate
Min.	2.82	5.28	4.87	4.79	11.61	6.00	21.57	5.09	39.19	3.04	83.46	1.76	234.89	1.24
Q_1	2.84	5.27	4.89	4.78	11.66	5.96	21.66	5.07	39.30	3.02	83.79	1.76	235.95	1.24
Median	2.85	5.24	4.91	4.77	11.69	5.95	21.70	5.06	39.40	3.02	84.15	1.76	236.56	1.24
Mean	2.87	5.01	4.92	4.74	11.74	5.92	21.79	5.04	39.53	3.01	84.31	1.76	237.70	1.25
Q_3	2.87	5.20	4.94	4.75	11.73	5.92	21.75	5.05	39.51	3.02	84.62	1.76	237.46	1.24
Max.	2.99	2.79	5.08	3.96	12.61	5.72	22.98	4.34	40.87	2.89	87.94	1.79	256.58	1.31
SD	0.04	0.04	0.04	0.03	0.20	0.09	0.30	0.06	0.37	0.03	0.85	0.02	4.24	0.02

Table 8. Optimal Solutions for Low Demand

Number of Clients	1,000		2,000		4,000		8,000	
Number of Vehicles	1,000	5,000	1,000	5,000	1,000	5,000	1,000	5,000
Clients	931	1,000	1,518	2,000	2,053	3,912	2,388	6,883
Increase in Clients	7.41%		31.75%		90.55%		188.23%	
Profit (R$)	79,308	84,661	133,877	172,360	182,993	338,187	217,231	605,970
Increase in Profits	6.75%		28.75%		84.81%		178.95%	
Vehicles	543	600	779	1,093	907	1,971	972	3,251
Increase in Vehicles	10.50%		40.31%		117.31%		234.47%	

Table 9. Optimal Solutions for High Demand

Number of Clients	16,000		32,000		64,000	
Number of Vehicles	1,000	5,000	1,000	5,000	1,000	5,000
Clients	2,596	9,972	2,664	11,812	2,729	12,897
Increase in Clients	284.13%		343.39%		372.59%	
Profit (R$)	237,779	886,743	248,519	1,067,651	254,744	1,189,949
Increase in Profits	272.93%		329.61%		367.12%	
Vehicles	987	4,205	989	4,685	999	4,915
Increase in Vehicles	326.04%		373.71%		391.99%	

raising profits and increasing the number of served clients when demand is at least 16,000 clients.

Simulating different carsharing modalities, such as one-way and free-floating would probably outcome different points of saturation. That difference will also happen when including the vehicle relocation tasks, and considering electric vehicles with constraints of time waiting until batteries be charged enough. The proposed formulations can be adapted for those wider and more complex scenarios, being able to also simulated and optimize the carsharing fleet-size in a computationally feasible run time. The following section presents the conclusion.

5. Conclusion

Carsharing services, together with other shared mobility products, are consistently changing the way people move in the city. The provision of better and cheaper products are enabling those services to emerge by benefiting more people with each passing day. In order to keep providing good services for low prices, tasks such as simulation and optimization must become routine for mobility enhancement companies.

Therefore, the development of computationally efficient methods for carsharing optimization is needed to simulate even bigger demand scenarios. This work proposed and compared two Mixed-Integer Linear Programming formulations for carsharing vehicle fleet-sizing in São Paulo, Brazil. Analysis of run time and of the optimal solutions were presented, varying the number of clients simulated and the maximum number of available vehicles and parking slots. The formulation without the Big-M method was shown to be faster and with more stable run times than the formulation using the Big-M.

According with the optimal solutions found, the number of served clients, earned profits and used vehicles started to saturate with demand of 16,000 clients per day. That saturation indicates that is needed more than 5,000 vehicles and parking slots for significantly increasing the number of served clients, and consequently, raising the company's profits. However, it is possible that only offering the round-trip modality does not attract a high demand of clients all days, making the carsharing company to also offer less restrict modalities. Wider and more more complex scenarios regarding different carsharing modalities would probably outcome different saturation points. Those simulations could also be performed in a computationally feasible run time using, as basis, the proposed formulation without Big-M.

As future work, we suggest to evaluate the impact of the blocks of constraints in the run times and memory needed through different scenarios. Another future work consists in proposing formulations based on electric vehicles and their use dynamics, which require longer times to charge the batteries. Also, the impact of time for charging batteries could be evaluated for carsharing services that are not station-based, such as the free-floating modality. In those cases, charging batteries can become an issue since clients can finish their rentals in places without a charging spot, not recharging the vehicle for the next client. Finally, we suggest, as future work, to evaluate even bigger scenarios (maybe using distributed computing), also regarding the one-way and free-floating modalities with vehicles relocation task, and considering the client's walking tolerance as a variable while looking for an available vehicle or charging spot.

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