GRAVITY ASSIST MANEUVERS APPLIED ON INTERPLANETARY TRAJECTORIES

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ABSTRACT

In this work, the problem of optimization of interplanetary trajectories with minimum fuel consumption, but with a time limit is studied. A methodology known as the Patched Conics was used, where the trajectory is divided into three parts: 1) departure phase, inside of the sphere of influence of the departure planet, 2) heliocentric phase, during the journey between the planets; 3) arrival phase, inside the sphere of influence of the arrival planet. Furthermore, was considered the possibility of gravitational assisted maneuvers (swing-by) to reduce the fuel consumption. In this case the full trajectory would be divided into more parts, depending on the number of maneuvers. Therefore, the goal of this work is to find a combination of conical trajectories, using gravitational assisted maneuvers, which perform the transfer from a nearby of the departure planet to the vicinity of the arrival planet, spending a minimal fuel with minimal time for the journey. Considering the minimization of the time, the solution cannot be the solution of minimum fuel consumption, because the minimization of time and the minimization of the fuel are conflicting objectives. Thus, a multi-objective problem must be solved. Hence, was used a methodology based on the Non Inferiority Criterion (Pareto[1], 1909) and the Smallest Loss Criterion (Rocco[2] et al. 2003), capable of considering multiple objectives simultaneously, without reducing the problem to the case of optimizing a single objective as occur in the most methods found in the literature. A mission to Pluto, similar to the NASA's New Horizons Mission, was studied considering gravitational assisted maneuvers in Jupiter and Saturn. Simulating the trajectories and the maneuvers using the Transfer Trajectory Design Programs (Sukhanov[3], 2004), several possibilities were analyzed for many combinations of fuel consumption, time of departure, time of arrival and planet used for the swing-by. Then, using the Multi-Objective Optimization Program (Rocco[4], 2002) the problem was solved seeking the best combination. The results can provide a good assistance for the mission analysis reducing the cost and time.

THE MULTI-OBJECTIVE PROBLEM

An optimization problem can have more than one objective, and in this case the objectives are normally conflicting. The classic methods are not recommended on these cases, because when one objective is optimized the others are no longer optimal. To solve this kind of problem a multi-objective optimization method can be used, this will lead to a sub optimal solution for each of the objectives separately. Most of the studies involving multi-objective optimization were developed in areas like economy, sociology and psychology. However, there are a lot more applications for multi-objective optimization, for example, engineering.

According to Cohon (Cohon[5], 1978), the static optimization of problems with one objective can be defined in the following way:

$$
\begin{array}{ll}
\text{Maximize} & \mathrm{Z}(\boldsymbol{x}) \ \text{ with relation to } \boldsymbol{x} \in \boldsymbol{R}^n \\
\text{Subject to} & \mathrm{g}_i(\boldsymbol{x}) \le 0 \qquad i = 1, 2, \ldots, m \\
 & \boldsymbol{x} \ge 0 \\
\text{Given} & \mathrm{Z}(\cdot),\ g_i(\cdot)
\end{array} \tag{1}
$$

or

$$
\begin{array}{ll}
\text{Maximize} & \mathrm{Z}(\boldsymbol{x}) \ \text{ with relation to } \boldsymbol{x} \in \boldsymbol{R}^n \\
\text{Subject to} & \boldsymbol{x} \in \boldsymbol{F}_d \\
\text{Given} & \mathrm{Z}(\,.\,),\ \boldsymbol{F}_d
\end{array} \tag{2}
$$

where $\boldsymbol{F}_d$ is the feasible area of the decision space, defined by:

$$\boldsymbol{F}_d = \{\boldsymbol{x} \in \boldsymbol{R}^n \mid \mathrm{g}_i(\boldsymbol{x}) \le 0, i = 1, 2, \ldots, m; \boldsymbol{x} \ge 0\} \tag{3}$$

The multi-objective problem can be defined by:

$$
\begin{array}{ll}
\text{Maximize} & \mathbf{Z}(\boldsymbol{x}) = [\mathrm{Z}_1(\boldsymbol{x}), \mathrm{Z}_2(\boldsymbol{x}), \ldots, \mathrm{Z}_p(\boldsymbol{x})] \\
\text{Subject to} & \boldsymbol{x} \in \boldsymbol{F}_d
\end{array} \tag{4}
$$

Therefore, in this case, the objective function, is a vector with dimension p.

In problems of one-dimensional optimization (when we have one objective), the possible solutions ($\boldsymbol{x} \in \boldsymbol{F}_d$) can be compared by means of the objective function, that is, given two solutions $\boldsymbol{x}^1$ and $\boldsymbol{x}^2$ we can compare $Z(\boldsymbol{x}^1)$ with $Z(\boldsymbol{x}^2)$ and determine the optimal solution so that $\boldsymbol{x} \in \boldsymbol{F}_d$ doesn't exist such that $Z(\boldsymbol{x}) > Z(\boldsymbol{x}^*)$. In problems of multi-dimensional optimization (multi-objective problem), in general, it is not possible to compare all the possible solutions because the comparison on the basis of one objective can be contradicted with the comparison based on another objective. Namely, supposing that:

$$\mathbf{Z}(\boldsymbol{x}^1) = [Z_1(\boldsymbol{x}^1), Z_2(\boldsymbol{x}^1)]$$
$$\mathbf{Z}(\boldsymbol{x}^2) = [Z_1(\boldsymbol{x}^2), Z_2(\boldsymbol{x}^2)] \tag{5}$$

$\boldsymbol{x}^1$ is better than $\boldsymbol{x}^2$ if and only if:

$$Z_1(\boldsymbol{x}^1) > Z_1(\boldsymbol{x}^2) \text{ and } Z_2(\boldsymbol{x}^1) \geq Z_2(\boldsymbol{x}^2) \tag{6}$$

or

$$Z_1(\boldsymbol{x}^1) \geq Z_1(\boldsymbol{x}^2) \text{ and } Z_2(\boldsymbol{x}^1) > Z_2(\boldsymbol{x}^2) \tag{7}$$

If $Z_1(\boldsymbol{x}^1) > Z_1(\boldsymbol{x}^2)$ and $Z_2(\boldsymbol{x}^1) < Z_2(\boldsymbol{x}^2)$ we cannot conclude anything regarding $\boldsymbol{x}^1$ and $\boldsymbol{x}^2$, that is, $\boldsymbol{x}^1$ e $\boldsymbol{x}^2$ cannot be compared.

Pareto Method (Non Inferiority)

On this method the optimal solution is one of the candidates in the group of possible solutions that are considered equally optimal. The Pareto method is not able to point only one solution for a multi-objective problem. According to Pareto, any candidate that belongs to this group could be chosen as a solution for the multi-objective problem, in other words, the degree of optimality would be the same for any solution in this group. Therefore, the solution choice would be made by a specialist capable of analyzing the gains and losses for each candidate of solution. So, once again, the solution for the problem will end up being individual for each specialist and for each case.

To select the group of Pareto's solutions, it's necessary to use an algorithm that will systematically make a comparison between the candidates. A solution *x* can only be considered optimal for a certain group of objectives, if there is not a solution *y* better in all objectives. The solution *x* is called non-inferior or non-dominated. Then, a solution *x* is non-dominated in case there is not a solution *y,* such that:

$$Z(y) \geq Z(x) \tag{8}$$

or

$$Z_k(y) \geq Z_k(x),\ k=1, 2,\ldots, p \tag{9}$$

If *y* exists, *x* is considered a dominated solution or inferior, and then it can't be an optimal Pareto's solution. A solution dominates the other if it's better in all objectives. It can be said that any pair of solutions in the group of the non-dominated solutions must be non-dominated one over another. And any dominated solution must be dominated by at least one solution of the group of the non-dominated solutions. The curve that contains the non-dominated solutions in known as Pareto frontier or Pareto set.

According to Kuhn-Tucker[6] (1951), if *x* is a non-dominated solution, then there must be multipliers $u_i \geq 0$, $i= 1, 2,\ldots, m$ and $w_k \geq 0$, $k= 1, 2,\ldots, p$ such that:

$X \in F_d$ (10)

$U_i g_i(x)=0,\ i= 1, 2,..., m$ (3.14)

$$\sum_{k=1}^{p} w_k \nabla Z_k(x) - \sum_{i=1}^{m} u_i \nabla g_i(x) = 0 \tag{11}$$

The first and third condition of Kuhn-Tucker are necessary conditions so that x is non-dominated. They are also sufficient in case that $Z_k(x)$ is concave for $k= 1, 2,.., p$, F_d is convex and $w_k > 0$ for every k.

<u>The Smallest Loss Criterion</u>

To solve the optimization problem, it would be convenient to use a methodology capable of finding a solution that covers all the objectives simultaneously. On the Pareto method the solution for the multi-objective problem with conflicting objectives can be chosen within a set of solutions considered non-dominated solutions or non-inferior. However, in this case we have a group of possibilities but not a final solution, since a solution must be chosen within this group. In any solution that is chosen there would be an objective being prioritized, then it would be necessary to stipulate weights for each objective, and this would be one more process of optimization. Besides, to apply it in engineering, the solution for an optimization process can't be determined in an uncertain and subjective way. Using a specialist makes the solution of the problem unpredictable and individual for that one specific specialist, making it difficult to reproduce the results by another specialist.

Actually, the specialist doesn't know which one is the optimal solution, the choice is based on particular criterion that will lead to a solution that can't be called optimal, since according to Pareto, there is a group of non-dominated solutions with the same degree of optimality. Therefore, can be said that any solution of that group could be chosen randomly, eliminating the need of a specialist.

There are some multi-objectives problems where all of the candidates for solution are non-dominated. In this case there's no use in determining a group of non-dominated solutions. There is still the need of finding a solution somehow. Therefore, it's necessary to apply another method.

The Smallest Loss Criterion (Rocco et al. 2000[7]; 2001[8]; 2003[2]; Rocco, 2002[4]) was elaborated in order to find one final solution that attends all objectives simultaneously in the best possible way, without the need of prioritizing any of them. Some applications of this method can be found on Rocco[9] et al. (2005a), Rocco[10] et al. (2005b) e Rocco[11] et al. (2005c).

The solution for a problem with n_conflicting objectives, where the goal is to optimize equally and simultaneously the objectives, must be the one that results on the smallest loss for each of the objectives, since there is no solution capable of optimizing the n objectives individually.

A way to obtain the smallest loss solution for a multi-objective problem would be to find the baricenter of a normalized n-dimensional figure, where on each vertex would be the optimal solution of each objective isolated. On a problem with three objectives, for example, the smallest loss solution would be at the center of a normalized triangle. So, for a problem with *n* objectives, the smallest loss solution would be at the center of this normalized *n*-dimensional figure. Figure *1*shows the example of a three conflicting objectives:

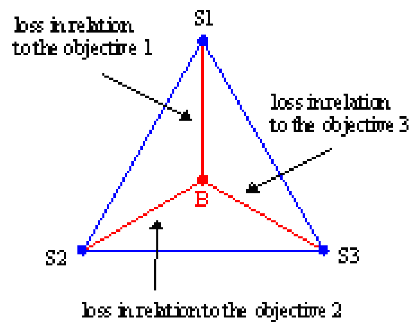

Fig 1: Smallest loss for three conflicting objectives

On this example, *S1, S2* e *S3* are the optimal solutions for each one of the objectives separately. *B* is the baricenter of the figure where each objective would have the smallest loss considering all the objectives together. Therefore, the distance between *S1* and *B* represents the smallest loss for the objective 1, the distance between *S2* and *B* for the objective 2 and the distance between *S3* and *B* the smallest loss for the objective 3. Then, according to figure *1*, the best solution for the multi-objective problem, considering all the objectives equally, would be at the center of the triangle.

GRAVITY ASSIST MANEUVERS

The gravity assist maneuvers (swing-by), for example, can be used to reduce fuel consumption on interplanetary missions. The smaller the ΔV, which is the change in velocity, the smaller will be the fuel cost. It also can reduce the duration of a mission.

The first time this maneuver was applied on a real mission was in 1974, when was launched the Mariner 10 probe, with swing-by on Venus and Mercury. It’s possible to make a sequential swing-by on different bodies on the same mission, so that the velocity gain can be even higher. On the Voyager mission, for example, the swing-by was made on Jupiter, Saturn, Uranus and Neptune, gaining more energy on each planet until there was enough energy to leave the solar system (Kohlhase[12] & Penzo, 1977). In 2006 the New Horizon spacecraft was launched with the purpose of studying Pluto and its moons (Guo[13] & Farquhar, 2006). On February of 2007 New Horizons made a swing-by on Jupiter and will arrive in Pluto on 2015.

The swing-by maneuvers can also be used for other purposes: on the Ulisses mission, in 1985, this maneuver was used to modify the orbital plane inclination of the probe; the use of consecutive swing-bys on the moon to obtain the geometry of the orbits desired, like satellites used to study the solar phenomenon.

TRAJECTORIES SIMULATION

An interplanetary trajectory program (Sukhanov[3], 2004) was used to simulate missions to Pluto with minimum fuel consumption based on the *patched conic* method. To obtain the minimum fuel consumption the gravity assist maneuvers (swing-by) were applied on Jupiter and Saturn. The program was written in Fortran language and can be used to generate launch and swing-by windows; optimum transfer trajectories for each day of the launch window, with the ΔV, mission duration, trajectory parameters; generate trajectories with restrictions such as total time of transfer; generate graphs for the trajectories that were obtained.

Patched Conics Method

A methodology called *patched conics* (Broucke[14], 1988) was used, in which the trajectory is divided into parts. The first part is the planetocentric one, inside the sphere of influence of the origin planet; the second is the heliocentric part, where the spacecraft is traveling from one planet to another; the third part is a planetocentric part, inside the sphere of influence of the destination planet.

Considering the variables: M_1 as being a massive body in the center of the cartesian system, M_2 a smaller body, that could be a planet or a satellite of M_1; M_3 a body with an infinitesimal mass around M_1, going towards M_2. The relative velocities are: V_2, is the velocity related to M_1; V_∞^- e V_∞^+ are the velocity vectors of the spacecraft related to M_2 before and after the encounter respectively; And the angles: δ, which is half of the angle between V_∞^- e V_∞^+; ψ, the angle between the periapsis line and the M_1-M_2 line. The variable r_p is the distance of maximum approximation when there is the encounter of M_2 and M_3.

The expression for δ can be obtained using the theory of hyperbolic orbits, given by:

$$\operatorname{sen}(\delta) = \frac{1}{1 + \frac{r_p V_\infty^2}{\mu_2}} , \tag{12}$$

where $\mu_2 = M_2 = Gm_2$, and G being the gravitational constant.

The spacecraft enters the sphere of influence of M_2 after leaving the Keplerian orbit around M_1, and from that on the effects of M_1 can be neglected. The velocities of the spacecraft before and after the encounter with M_2 are given by equations *(13)* and *(14)* respectively:

$$\vec{V}_\infty^- = \vec{V}_i - \vec{V}_2 \tag{13}$$

$$\vec{V}_\infty^+ = \vec{V}_o - \vec{V}_2 \tag{14}$$

The difference between the inertial velocity before and after the swing-by is obtained using the equations *(13)* and *(14):*

$$\vec{V}_\infty^+ - \vec{V}_\infty^- = \Delta\vec{V} = \vec{V}_o - \vec{V}_i \tag{15}$$

According to the vector diagram, the magnitude of the velocity variation is given by:

$$\Delta V = \left|\Delta\vec{V}\right| = 2\left|\vec{V}_\infty\right|\sin(\delta) = 2V_\infty \sin(\delta) \tag{16}$$

The components X and Y of the velocity increasing are:

$$\Delta\dot{X} = -2V_\infty \sin(\delta)\cos(\psi) \tag{17}$$

$$\Delta\dot{Y} = -2V_\infty \sin(\delta)\sin(\psi) \tag{18}$$

The angular momentum C is given by:

$$C = X\dot{Y} - Y\dot{X} \tag{19}$$

Therefore, the angular momentum variation *ΔC* is:

$$\Delta C = X\left(\Delta\dot{Y}\right) + (\Delta X)\dot{Y} - Y\left(\Delta\dot{X}\right) - (\Delta Y)\dot{X} \tag{20}$$

Considering that the encounter is instantaneous, in other words, that $\Delta X = \Delta Y = 0$, $t = 0$ e $Y = 0$, the angular momentum variation will be:

$$\Delta C = X\Delta\dot{Y} \tag{21}$$

Resulting in:

$$\omega\Delta C = -2V_2 V_\infty \operatorname{sen}(\delta)\operatorname{sen}(\psi) \tag{22}$$

It's possible to obtain the energy variation subtracting the energy after the swig-by E_+ from the energy before $E\_$:

$$\Delta E = E_+ - E_- = \frac{1}{2}\left[\left(\dot{X} + \Delta\dot{X}\right)^2 + \left(\dot{Y} + \Delta\dot{Y}\right)^2\right] - \frac{1}{2}\left(\dot{X}^2 + \dot{Y}^2\right) \tag{23}$$

Figure 1 shows the variables on a swing-by maneuver (Prado[15]):

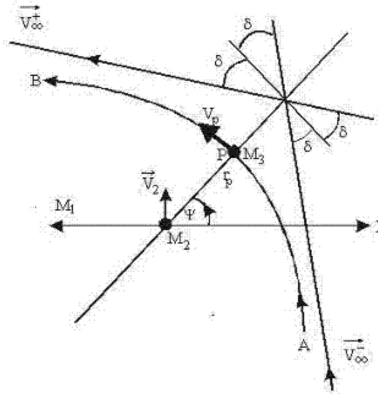

Fig 2: Swing-by maneuver

RESULTS OBTAINED WITH THE MULTI-OBJECTIVE PROGRAM

To resolve the multi-objective problem, three objectives were considered: the fuel consumption, in other words, the ΔV; the duration of the mission; the waiting time on Earth for launch. The reference day is January 1st of 2012. Therefore, for September 1st of 2012, the waiting time would be of eight months. The normalization was obtained using the maximum value of each objective.

On the search for the multi-objective solution, three possibilities were considered:
1-) Case I: using only the extreme non-dominated candidates: on this case, to obtain the baricenter of the figure, are considered only the three extreme non-dominated candidates. These candidates are those in which at least one of the objectives is optimal, in other words, one candidate has the objective *a* optimal, the second candidate has the objective *b*, and the third the objective *c* optimal. Therefore, the solution for the multi-objective problem is found using the baricenter of the extreme non-dominated candidates.

2-) Case II: using all the candidates: on this case the baricenter of the figure is obtained considering all the candidates, and then the search for the smallest loss solution is made.

3-) Case III: using the utopian solution: the utopian solution would be the ideal solution for the problem, where each of the three objectives is optimal. However, because the objectives are conflicting, this solution is not possible. Then on this case, the optimal solution is the closest one to the utopian solution.

Swing-by on Jupiter in September of 2012 and Pluto Arriving Window from 2025 to 2030

This simulation corresponds to month September of 2012, where the spacecraft makes a swing-by on Jupiter, with arriving window on Pluto from 2025 to 2030. The figure *5* shows all the candidates for solution considering the three objectives to be optimized, where *an* is the fuel consumption, *bn* the duration of the mission, and *cn* the waiting time for launch. Figures *6*, *7* and *8* show for case I, case II and case:

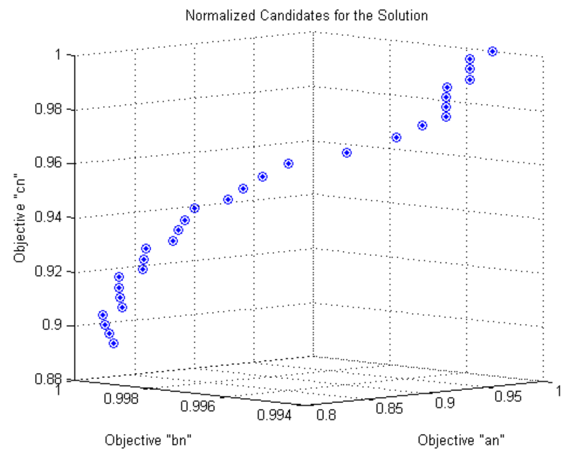

Fig 3: Normalized candidates for solution

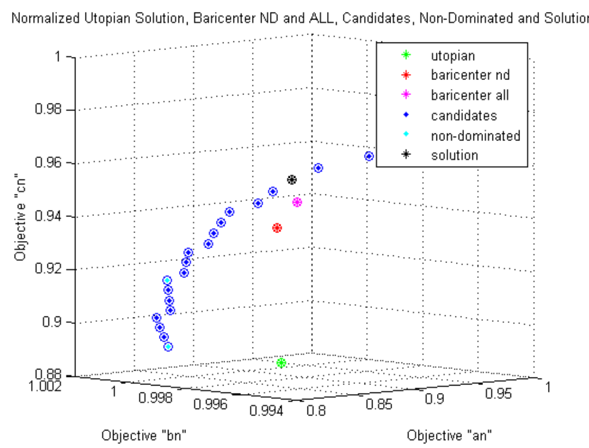

Fig 4: solution for case I, candidates, normalized non-dominated candidates, normalized utopian solution, normalized baricenter of the non-dominated, and all the candidates

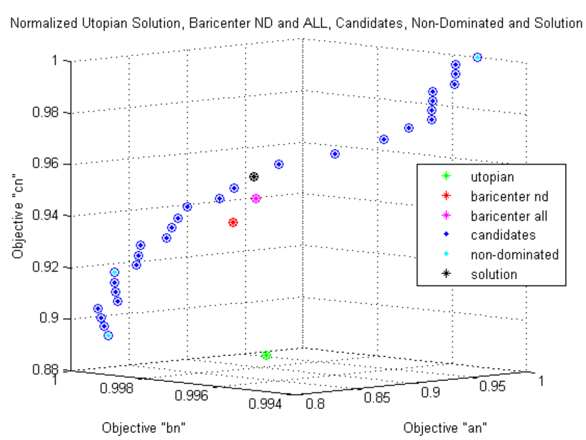

Fig 5: solution for case II, candidates, normalized non-dominated candidates, normalized utopian solution, normalized baricenter of the non-dominated, and all the candidates

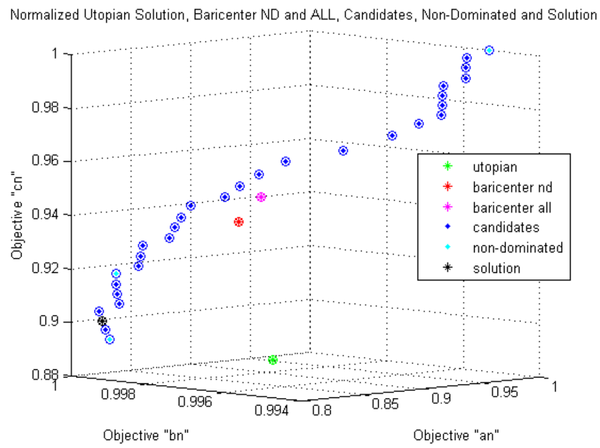

Fig 6: solution for case III, candidates, normalized non-dominated candidates, normalize utopiansolution, normalized baricenter of the non-dominated, and all the candidates

Table 2: Solutions for case I, II, and III

Table 2	**Case I (18)**	**Case II (19)**	**Case III (2)**
Δv (km/s)	7.831	7.831	7.355
duration (years)	17.07	17.07	17.11
waiting T (days)	317	317	301

<u>Swing-by on Saturn in September of 2012 and Pluto Arriving Window from 2025 to 2030</u>

This simulation corresponds to month September of 2012, where the spacecraft makes a swing-by on Saturn, with arriving window on Pluto from 2025 to 2030. The figure *5* shows all the candidates for solution considering the three objectives to be optimized, where *an* is the fuel consumption, *bn* the duration of the mission, and *cn* the waiting time for launch. Figures *6*, *7* and *8* show for case I, case II and case:

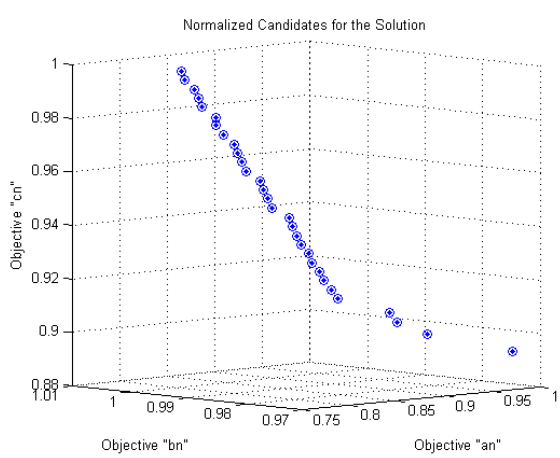

Fig 7: Normalized candidates for solution

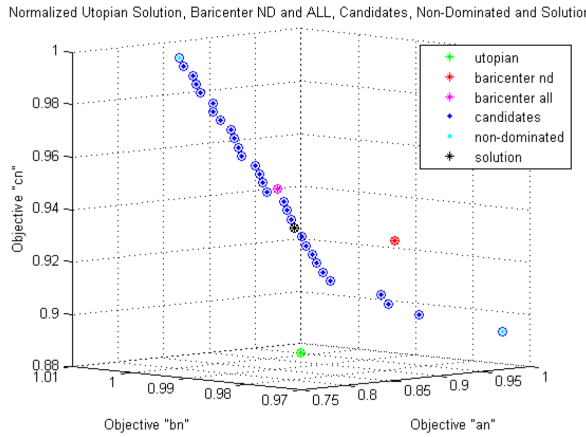

Fig 8: solution for case I, candidates, normalized non-dominated candidates, normalized utopian solution, normalized baricenter of the non-dominated, and all the candidates

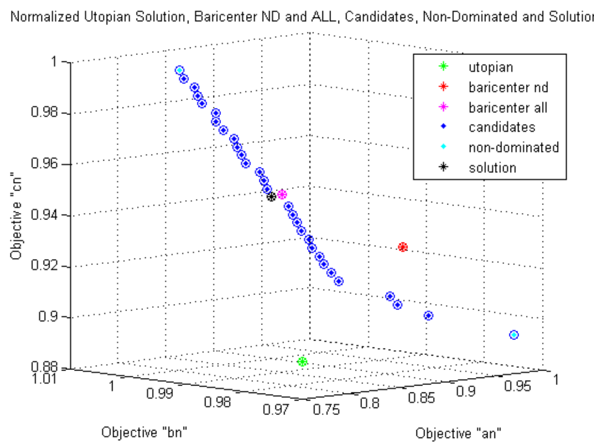

Fig 9: solution for case II, candidates, normalized non-dominated candidates, normalized utopian solution, normalized baricenter of the non-dominated, and all the candidates

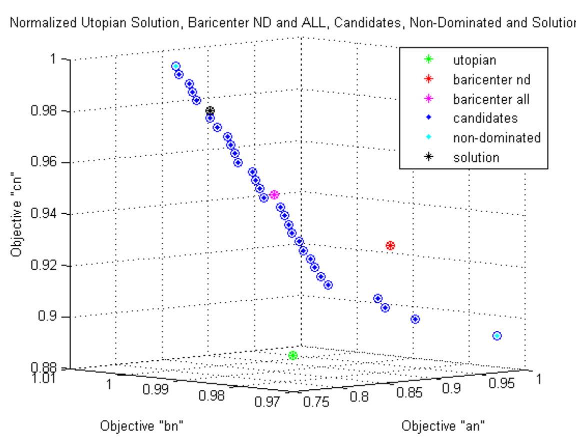

Fig 10: solution for case III, candidates, normalized non-dominated candidates, normalize utopiansolution, normalized baricenter of the non-dominated, and all the candidates

Table 2: Solutions for case I, II, and III

Table 2	Case I (11)	Case II (15)	Case III (24)
Δv (km/s)	23.091	22.319	20.603
duration (years)	17.08	17.08	17.05
waiting T (days)	310	314	323

CONCLUSION

A group of simulations to Pluto were obtained using swing-by maneuvers on Jupiter and Saturn. These simulations were applied on a multi-objective optimization program, in which the goal was to optimize fuel consumption, duration of the mission, and waiting time for launch. The launch window was the month of September of 2012, and the arriving windows were from 2025 to 2030.

Analyzing the simulations it was possible to verify the importance of the epoch chosen for the mission, because the parameters of the trajectory depend on the position that the planets are on that specific date.

The planet for the swing-by was selected according to the position it would be on the dates that were chosen for the simulations. It would not be appropriate to use Uranus, for example, because this planet is out of the configuration needed on the launch and arriving period determined for the simulations. This geometric configuration has a big influence on the objectives that we want to optimize.

The multi-objective optimization program used to search for the optimal solutions was developed based on the smaller loss method. Different from other multi-objective methods, it is possible to find one final solution and not a group of feasible solutions (candidates to the solution for the multi-objective problem), that in most cases, leads to the necessity of considering different weights for the objectives, since it is not possible to chose only one solution that is better in all objectives. This group of solutions considered equally good, or non-dominated solutions according to Pareto, often includes a very large number of candidates. In some cases, as on this study, practically all the feasible solutions belong to Pareto frontier, in other words, all the candidates are non-dominated. Therefore, for the cases which all or most of the candidates are non-dominated, determining the Pareto frontier is not the best way to look for the best solution, since it would not limit the number of candidates. Such cases are common when the objectives are diametrically opposed, for example, minimizing time and fuel consumption in orbital maneuvers. So, to select only one alternative among all the candidates for the solution of the multi-objective problem, another method must be applied. With the smaller loss criterion it is possible to find a solution that attends all the objectives simultaneously without the need to prioritize any of them.

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