A Comparison Between Numerical and Analytical Methods to Predicted Orbits of Particles

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Abstract— The idea of the present paper is to study the swing-by maneuver between one of the planets of the Solar System and a cloud of particles. The main point considered in the present paper is to compare the results predicted by the analytical and the numerical approach to solve this problem. So, the results will be concentrated in figures that show the difference in the results obtained from both methods in several situations. A cloud of particle can be obtained when a fragmented comet crosses the orbit of a planet like Jupiter, Saturn, etc. For the numerical study we assumed that the restricted three-body problem is a good model for the system. So, the planet e Sun are in circular orbits around the center of mass and a cloud of particles is moving under the gravitational attraction of these two primaries. The motion is assumed to be planar for all the particles. For the analytical study, the dynamics is given by the "patched-conic" approximation, which means that a series of two-body problems are used to generate the equations that describe the problem. The main obejctive is to understand the change of the orbit of this cloud of particles after the close approach with the planet by both methods. It is also assumed that all the particles that belong to the cloud have semi-major axis a ± da and eccentricity e ± de before the close approach with the planet. It is desired to known those values after the close approach, as predicted by both methods.

Key-Words: astrodynamics, artificial satellites, orbital dynamics, swing-by.

1 Introduction

The close approach between a particle and a planet modifies the velocity, energy and angular momentum of the particle with respect to the Sun. There are many important applications of this phenomenon in astronautics, like the Voyager I and II that used successive close encounters with the giant planets to make a long journey to the outer Solar System; the Ulysses mission that used a close approach with Jupiter to change its orbital plane to observe the poles of the Sun, etc. References [1] to [26] show some results of this type.

In the present paper we study the close approach between a planet and a cloud of particles. It is assumed that the dynamical system is formed by two main bodies (the Sun and one planet) that are in circular orbits around their center of mass and a cloud of particles that is moving under the gravitational attraction of the two primaries. To study this motion, two different techniques are used: 1) a numerical study based in the numerical integrations of the equations of motion given by the restricted three-body problem and; 2) a motion described by the "patched-conic" approximation, that is a series of two-body problems between that particle and the Sun, between the particle and the planet and then between the particle and the Sun again.

The standard canonical system of units is used in both cases and it implies that the unit of distance is the distance between the two primaries and the unit of time is chosen such that the period of the orbit of the two primaries is 2π.

The main objective is to study the change of the orbit of each element of this cloud of particles after the close approach with the planet. It is assumed that all the particles that belong to the cloud have semi-major axis a ± da and eccentricity e ± de before the close approach with the planet. It is desired to known those values after the close approach.

Among the several sets of initial conditions that can be used to identify uniquely one swing-by trajectory, a modified version of the set used in the papers written by Broucke and Prado [6], [7] and [8] is used here. It is composed by the following three variables:

1) V_p, the velocity of the spacecraft at periapse of the orbit around the secondary body;

2) The angle γ, that is defined as the angle between the line M1-M2 (the two primaries) and the direction of the periapse of the trajectory of the spacecraft around M2;

3) r_p, the distance from the spacecraft to the center of M2 in the moment of the closest approach to M2 (periapse distance).

2 Analytical Equations

This section will briefly describe the orbital change of a single particle subjected to a close approach with the planet under the "patched-conics" model, in order to build a set of analytical equations to solve the problem of calculating the effect of the swing-by in the orbit of the particles. It is assumed that the particle is in orbit around the Sun with given semi-major axis (a) and eccentricity (e). The periapse distance (r_p) is assumed to be known. The first step is to obtain the energy (EB) and angular momentum (CB) of the particle before the swing-by. They are given by:

$$EB = -\frac{(1-\mu_J)}{2a} \qquad (1)$$

and

$$CB = \sqrt{(1-\mu_J)a(1-e^2)} \qquad (2)$$

Then, it is possible to calculate the magnitude of the velocity of the particle with respect to the Sun in the moment of the crossing with Jupiter's orbit (Vi), as well as the true anomaly of that point (θ). They come from:

$$V_i = \sqrt{(1-\mu_J)\left(\frac{2}{r_{SJ}} - \frac{1}{a}\right)} \qquad (3)$$

and

$$\theta = \cos^{-1}\left[\frac{1}{e}\left(\frac{a(1-e^2)}{r_{SJ}} - 1\right)\right] \qquad (4)$$

Next, it is calculated the angle between the inertial velocity of the particle and the velocity of Jupiter (the flight path angle d), as well as the magnitude of the velocity of the particle with respect to Jupiter in the moment of the approach. They are given by (assuming a counter-clock-wise orbit for the particle):

$$\gamma = \tan^{-1}\left[\frac{e\sin\theta}{1+e\cos\theta}\right] \qquad (5)$$

$$V_\infty = \sqrt{V_i^2 + V_2^2 - 2V_i V_2 \cos\gamma} \qquad (6)$$

The angle β is given by:

$$\beta = \cos^{-1}\left[-\frac{V_i^2 - V_2^2 - V_\infty^{-2}}{2V_2 V_\infty^-}\right] \qquad (7)$$

Those information allow us to obtain the turning angle (2δ) of the particle around Jupiter, from:

$$\delta = sin^{-1}\left(1 \bigg/ 1 + \frac{r_p V_\infty^2}{\mu_J}\right) \qquad (8)$$

The angle of approach has two values, depending if the particle is passing in front (when we call Solution 1) or behind (when we call Solution 2) the planet. These two values will be called ψ1 and ψ2. They are obtained from:

$$\psi_1 = \pi + \beta + \delta \qquad (9)$$

and

$$\psi_2 = 2\pi + \beta - \delta \qquad (10)$$

The correspondent variations in energy and angular momentum are obtained from the equation

$$\Delta C = \Delta E = -2 V_2 V_\infty \, sen\,\delta \, sen\,\psi \qquad (11)$$

By adding those quantities to the initial values we get the values after the swing-by. Finally, to obtain the semi-major axis and the eccentricity after the swing-by it is possible to use the equations

$$a = -\frac{\mu}{2E}, \quad e = \sqrt{1 - \frac{C^2}{\mu a}} \qquad (12)$$

3 Numerical Algorithm

For the numerical simulations, the equations of motion for the spacecraft are assumed to be the ones given by the three-dimensional restricted circular three-body problem. The standard dimensionless canonical system of units is used, which implies that: the unit of distance is the distance between M_1 and M_2; the mean angular velocity (ω) of the motion of M_1 and M_2 is assumed to be one; the mass of the smaller primary (M_2) is given by $\mu = m_2/(m_1 + m_2)$ (where m_1 and m_2 are the real masses of M_1 and M_2, respectively) and the mass of M_2 is (1-μ); the unit of time is defined such that the period of the motion of the two primaries is 2π and the gravitational constant is one.

There are several systems of reference that can be used to describe the three-dimensional restricted three-body problem [8]. In this paper the rotating system is used.

In the rotating system of reference, the origin is the center of mass of the two massive primaries. The horizontal axis (x) is the line that connects the two primaries at any time. It rotates with a variable angular velocity in a such way that the two massive primaries are always on this axis. The vertical axis (y) is perpendicular to the (x) axis. In this system, the positions of the primaries are: $x_1 = -\mu$, $x_2 = 1 - \mu$, $y_1 = y_2 = 0$.

In this system, the equations of motion for the massless particle are [27]:

$$\ddot{x} - 2\dot{y} = x - (1-\mu)\frac{x+\mu}{r_1^3} - \mu\frac{x-1+\mu}{r_2^3} \qquad (13)$$

$$\ddot{y} + 2\dot{x} = y - (1-\mu)\frac{y}{r_1^3} - \mu\frac{y}{r_2^3} \qquad (14)$$

$$\ddot{z} = -(1-\mu)\frac{z}{r_1^3} - \mu\frac{z}{r_2^3} \qquad (15)$$

where r_1 and r_2 are the distances from M_1 and M_2.

A numerical algorithm to solve the problem has the following steps:

1) Arbitrary values for the three parameters r_p, V_p, α, β and γ are given;

2) With these values the initial conditions in the rotating system are computed. The initial position is the point (X_i, Y_i, Z_i) and the initial velocity is (V_{Xi}, V_{Yi}, V_{Zi}), given by equations: Position:

$$x_i = r_p \cos\beta \cos\alpha$$
$$y_i = r_p \cos\beta \sin\alpha$$
$$z_i = r_p \sin\beta$$

Velocity:

$$V_{xi} = -V_p \sin\gamma \sin\beta \cos\alpha - V_p \cos\gamma \sin\alpha$$
$$V_{yi} = -V_p \sin\gamma \sin\beta \sin\alpha + V_p \cos\gamma \cos\alpha$$
$$V_{zi} = V_p \cos\beta \sin\gamma$$

3) With these initial conditions, the equations of motion are integrated forward in time until the distance between M_2 and the spacecraft is larger than a specified limit d. At this point the numerical integration is stopped and the energy (E_+) and the angular momentum (C_+) after the encounter are calculated;

4) Then, the particle goes back to its initial conditions at the point P, and the equations of motion are integrated backward in time, until the distance d is reached again. Then the energy (E_-) and the angular momentum (C_-) before the encounter are calculated. The criteria to stop numerical integration is the distance between the spacecraft and M_2. When this distance reaches the value d = 0.5 (half of the semimajor axis of the two primaries) the numerical integration is stopped. With this algorithm available, the given initial conditions (values of r_p, v_p, α, β, γ) are varied in any desired range and the effects of the close approach in the orbit of the spacecraft are studied.

4 Results

Both algorithm just described can now be applied to a cloud of particles passing close to a planet like Jupiter. The idea is to simulate a cloud of particles that have orbital elements given by: a ± da and e ± de. The goal is to map this cloud of particles to obtain the new distribution of semi-major axis and eccentricities after the swing-by using both algorithms and then to plot the differences between the two models, expressed by a percentage of the difference.

For the simulations, we used case $da = de = 0.001$, $r_p = 1.5$ R_J. Figures 1 to 4 show the results. It was assumed that a satellite explodes when passing by the periapsis in a given position. In those examples, this position is given by $\alpha = 30°$, $\beta = 45°$. Then, a reference value was used for the direction of the velocity: $\gamma = 60°$. The velocity at periapsis was assumed to be $v_p = 4.0$, expressed in canonical units. The vertical axis shows the difference between the results obtained between the two models considered, as a percentage difference. The horizontal axis shows the value of γ, in radians.
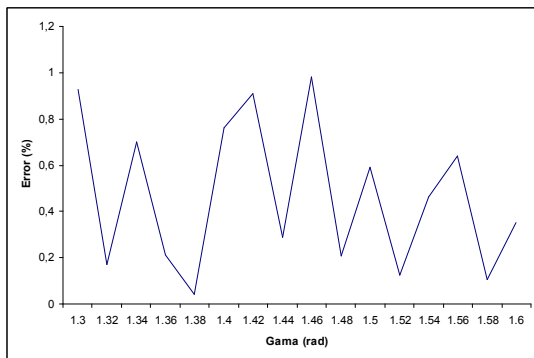

Fig. 1 – Differences for the Variation in Inclination (%) for $r_p = 1.5$ r_J and $v_p = 4.0$.
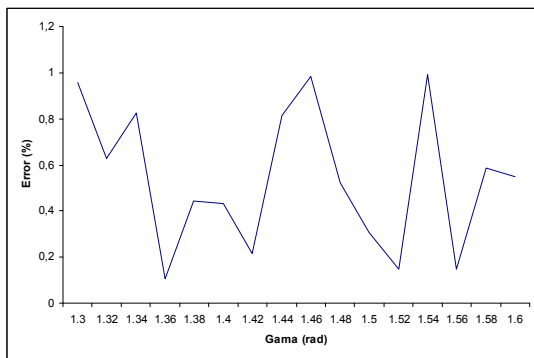

Fig. 2 – Differences for the Variation in Velocity (%) for $r_p = 1.5$ r_J and $v_p = 4.0$.
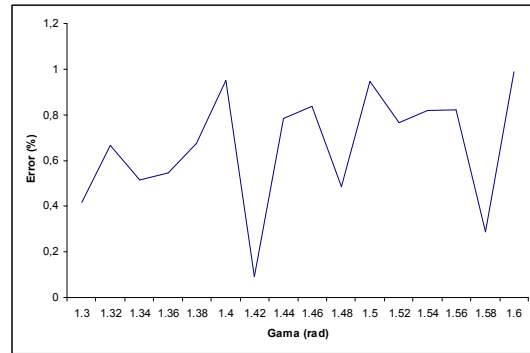

Fig. 3 – Differences for the Variation in Angular momentum (%) for $r_p = 1.5$ r_J and $v_p = 4.0$.
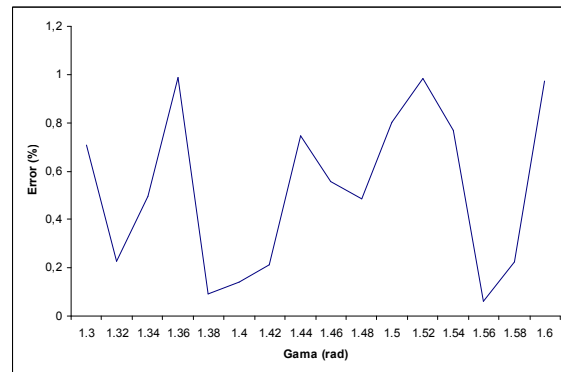

Fig. 4 – Differences for the Variation in Energy (%) for $r_p = 1.5$ r_J and $v_p = 4.0$.

5 Conclusions

The figures above allow us to get some conclusions. It shows that the analytical method is very good and accurate. The errors are always limited to a maximum of 1%. So, for any practical purpose, it is possible to use the analytical method as a very good approximation, that can be verified later by the numerical method.

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