Performance Comparison between Thrusters PPS1350 and the Phall 1 in Optimal Trajectories using Flybies and Solar Electric Propulsion

Denilson Paulo Souza dos Santos[1], Antonio Fernando Bertachini de Almeida Prado[1]
[1]National Institute for Space Research (INPE)
São José Dos Campos - São Paulo-12227-010 - Brazil
denilson@dem.inpe.br, prado@dem.inpe.br

Abstract: - The spacecraft propulsion system has passed for diverse evolutions, leaving combustion engines and arriving at ion propulsion. The future interplanetary missions will probably use the conventional chemical rockets to leave the sphere of influence of the Earth, and solar electric propulsion (SEP) to accomplish the other maneuvers of the mission. The optimization of maneuvers will be approached in interplanetary missions using solar electric propulsion and Gravity Assisted Maneuver attended to reduce the costs of the mission. Exploring the high specific impulse and the capacity of the electric propulsion to obtain a Gravity Assisted Maneuver with the Earth after a year of the departure of the spacecraft.
Shall be analyzed missions with thrusters PPS1350 and the Phall 1 for performance comparison. An indirect optimization method will be used in the simulations.

Key-Words: - Space Trajectories, Applied Mathematics, Astrodynamics, Celestial Mechanics, Space Trajectories, Flyby.

1 Introduction

The solar electric propulsion could be the best option for the transports of the future due to its high specific impulse when compared to the chemical propulsion. Electric propellants are being extensively used to assist the propulsion of terrestrial satellites for the maneuvers of orbit correction and as primary propulsion in missions toward other bodies of the solar system.

Both NASA and ESA have launched spacecrafts which used SEP (Solar Electric Propulsion) as the primary propulsion system; NASA's DS1 and ESA's Smart-1 to the moon to comet Borrelly.

Indirect optimization methods are suitable for the low thrust trajectories that are used in simulations. A finite force is applied during a finite interval of time and it is necessary to integrate the state equation along the time to know its effect. Several results exist in literature, starting with the works of Tsien (1953) and Lawden (1955). Other results and references can be found in Prado (1989), Prado and Rios-Neto (1993), Casalino and Colasurdo [1], Santos [2]. The most used method in this model is the so called "primer-vector theory", developed by Lawden (1953 and 1954) [4], [5], according to Prado [6], [7], Santos [8], [9]. In this paper, theory of optimal control is applied and a procedure based on the Newton Method to decide the boundary problems is developed. The Pontryagin's Maximum Principle (PMP) is used to maximize the

Hamiltonian associated to the problem and evaluates the optimal structure of the "switching function".

The spacecraft leaves the Earth's sphere of influence with a hyperbolic velocity whose optimal magnitude and the direction will be supplied by the optimization procedure. The initial mass is directly related to the magnitude of the hyperbolic velocity, assuming that a chemical thruster is used to leave a low Earth orbit (LEO). Out of the Earth's sphere of influence, the electric propellants is activate and the available power is proportional to the square of the distance from the sun; the propulsion is provided by one or two "PPS 1350 ion thrusters and Phall 1 (UNB)".

2 Description of the Problem

The spacecraft will be considered a point with variable mass m and trajectory will be analyzed using the patched-conics approach. The time required by the spacecraft to leave the Earth's sphere of influence is neglected and, in this formulation, only equations of motion in the heliocentric reference system will be considered. The spacecraft is influenced by the Sun gravitational acceleration $\vec{g}(r)$ and the propulsion system of the vehicle implements a thrust T. With this formulation, a maneuver of Earth flyby can be used to gain energy and velocity, that provokes a discontinuity in the relative state variables in the velocity.

The variables are normalized using the radius of the Earth's orbit, the corresponding circular velocity, and

the mass of the spacecraft in stationary orbit as values of reference.

The solar electric Propulsion will be considered, therefore, the available power and thrust varies with the square of the distance from the sun.

In the problem, the thrust is the only control during the heliocentric arcs, and it will be optimized to get the minimum consumption, that is measured by the final mass of the spacecraft. Since the thrust appears linearly in the equation of motion, a bang-bang control, that consists of alternating ballistic arcs with arcs of maximum thrust will be required. The trajectory is composed by a succession of ballistic arcs (zero-thrust) and arcs of maximum thrust, where the optimal direction will be supplied by the optimization procedure.

The boundary conditions are imposed in satisfactory way at the junctions between trajectory arcs.

The integration initiates when the spacecraft leaves the Earth's sphere of influence, at the position $\vec{r}_i = \vec{r}_\oplus(t_i)$ that coincides with the Earth is position, considering the velocity $\vec{v}_i$ free. The hyperbolic velocity is given by $\vec{v}_{\infty i} = \vec{v}_i - \vec{v}_\oplus(t_i)$, assuming that a rocket thruster is used to leave the Low Earth Orbit (LEO) with an impulsive maneuver; the vehicle mass on LEO is specified. The increment of velocity (ΔV) demanded to provide the hyperbolic velocity is $\Delta V = \sqrt{v_{\infty i}^2 + v_e^2} - v_c$, where v_e and v_c are the escape and circular velocity at the LEO radius [9].

The initial mass at the exit from the Earth's sphere of influence is,

$$m_i = a - bV_\infty - cV_\infty^2 \qquad (1)$$

where,

$\varepsilon(1-m_i)$ is the jettisoned mass of the exhausted motor, which is proportional to the propellant mass. The spacecraft intercepts the Earth and accomplishes Gravity Assisted Maneuvers (Santos et al., 2005) [8 - 9]. The position of the vehicle $\vec{r}_\pm = \vec{r}_\oplus(t_\pm)$ is constrained and the magnitude of the hyperbolic excess velocity $\vec{v}_{\infty\pm} = \vec{v}_\pm - \vec{v}_\oplus(t_\pm)$ is continuous $v_{\infty+}^2 = v_{\infty-}^2$ [2].

If the minimum height constraint on the *flyby* is requested, a condition on the velocity turn angle is added:

$$\vec{v}_{\infty+}^T \vec{v}_{\infty-} = -\cos(2\phi)v_{\infty-}^2 \qquad (2)$$

where,

$$\cos(\phi) = \frac{v_p^2}{\left(v_{\infty-}^2 + v_p^2\right)} \qquad (3)$$

v_p is the circular velocity at the low distances allowed for a planet.

$$\vec{v}_{\infty\pm} = \vec{v}_{i\pm} - \vec{v}_4 \qquad (4)$$

At the final point (subscript f), the position and velocity vectors of the spacecraft and the asteroid coincide,

$$r_f = r_A(t_f) \qquad (5)$$

$$v_f = v_A(t_f) \qquad (6)$$

The theory of optimal control provides the control law and necessary boundary conditions for optimality.

3 Optimization Procedures

The objective is to use the theory of optimal control to maximize the spacecraft final mass.
Dynamical equations are,

$$\dot{\vec{r}} = \vec{v}$$

$$\dot{\vec{v}} = \vec{g}(\vec{r}) + \frac{\vec{T}}{m} \qquad (7)$$

$$\dot{m} = -\frac{\vec{T}}{c}$$

Applying the theory of optimal control, the Hamiltonian function is defined as (Lawden, 1954) [3, 2]:

$$H = \vec{\lambda}_r^t \vec{v} + \vec{\lambda}_v^t(\vec{g} + \frac{\vec{T}}{m}) - \lambda_m \frac{\vec{T}}{c} \qquad (8)$$

An indirect optimization procedure is used to maximize the payload. According to Pontryagin's Maximum Principle the optimal controls maximize H. The nominal thrust T_o at 1 AU, and the electrical power are (Casalino, L. and Colasurdo, G., 2002) [1],

$$P_0 = \frac{T_0 c}{2\eta}$$

$$T_{Max} = \frac{T_0}{r^2} \qquad (9)$$

Optimal control theory provides differential equation for the adjoint equations of the problem (Euler-Lagrange).

Adjoint equations are,

$$\dot{\vec{\lambda}}_r^t = \vec{\lambda}_v^t \frac{\partial \vec{g}}{\partial \vec{r}} - S_f \frac{\partial T}{\partial \vec{r}} \qquad (10)$$

$$\dot{\vec{\lambda}}_v^t = -\vec{\lambda}_r^t \qquad (11)$$

$$\dot{\vec{\lambda}}_r^t = \lambda_v^t \frac{\vec{T}}{m^2} \qquad (12)$$

where, $G = \dfrac{\partial \vec{g}}{\partial \vec{r}}$.

Optimal control: thrust direction an magnitude are,

$$\vec{T} \;//\; \vec{\lambda}_v$$

$$H = \vec{\lambda}_r^t \vec{v} + \vec{\lambda}_v^t G + \vec{T}\left(\frac{\lambda_v}{m} - \frac{\lambda_m}{c'}\right) \qquad (13)$$

$$S_f = \frac{\lambda_v}{m} - \frac{\lambda_m}{c'}$$

where, c' - is the effective exhaust velocity of the rocket thruster;

$$T_{Max} = \begin{cases} \dfrac{T_0}{r^2} \to S_f > 0 \\[2mm] 0 \to S_f < 0 \end{cases} \qquad (14)$$

The necessary optimal conditions [5, 2].

$$\left(H_{j_-} + \frac{\partial \varphi}{\partial t_{j_-}} + \mu^t \frac{\partial \vec{\chi}}{\partial t_{j_-}}\right)\delta t_{j_-} = 0 \qquad (15)$$

$$\left(H_{j_+} - \frac{\partial \varphi}{\partial t_{j_+}} - \mu^t \frac{\partial \vec{\chi}}{\partial t_{j_+}}\right)\delta t_{j_+} = 0 \qquad (16)$$

$$\left(\lambda_{j_-}^t - \frac{\partial \varphi}{\partial \vec{x}_{j_-}} - \mu^t \frac{\partial \vec{\chi}}{\partial \vec{x}_{j_-}}\right)\delta \vec{x}_{j_-} = 0 \qquad (17)$$

$$\left(\lambda_{j_+}^t + \frac{\partial \varphi}{\partial \vec{x}_{j_+}} + \mu^t \frac{\partial \vec{\chi}}{\partial \vec{x}_{j_+}}\right)\delta \vec{x}_{j_+} = 0 \qquad (18)$$

Where:

$\vec{\chi}$: the vector collecting the constraining boundary conditions (see eq. 15 - 18)

$\varphi = m_f$

At the initial point:

1. $\vec{r}_0 = \vec{r}_\oplus$;

2. $m_o = 1 - bV_\infty - cV_\infty^2$

3. $(\vec{v}_0 - \vec{v}_\oplus)^2 = \vec{v}_{\infty 0}^2$;

4. Equations 16 and 18 provide optimal control with λ_{ro} and T_{ro} free;

5. the necessary condition optimal of the state is $\vec{\lambda}_{v0}$ (primer vector) be parallel to the hyperbolic velocity;

At flyby [1]:

1. the equations (15 and 16) are used to obtain the transversality conditions, that implicates in determining the arc time used;

2. at the equations (17 and 18) the $\vec{\lambda}_{vi}$ is parallel to the hyperbolic velocity, before and after of free flyby maneuver; the magnitude is continuous;

3. the states of Hamiltonian remain continuous through the *flyby* maneuvers;

4. when the minimum height constraint of the *flyby* is requested, a condition on the velocity turn angle is added (Eq. 2 and 3).

At the final point:

1. $\vec{\lambda}_{vf}$ is parallel to the hyperbolic velocity, $\vec{\lambda}_{rf}$ is parallel to the radius and $\vec{\lambda}_{rf}^t \vec{v}_f + \vec{\lambda}_{vf}^t \vec{g} = 0$;

2. the final values of $\vec{\lambda}_{mf}$ and H_f depends on the control model that was considered in the maneuver;

3. the adjoint variable $\vec{\lambda}_v$ is zero during the whole trajectory.

4 Numerical Analysis with Phall 1 (UNB)

The researchers of the Plasma Laboratory of the Physics Institute of the Brasilia University (UNB), since 2002, pledge in the study and development of a propellant that uses a plasma propulsion system produced by current Hall, based on Stationary Plasma Thrusters (SPT). They use permanent magnets with generating the magnetic field, reducing the electricity consumption.

The characteristics of the spacecraft propulsion system are:

1. the mass of the spacecraft with an altitude of 200 km in circular LEO is 2133.3 Kg;

2. specific impulse I_s = 1607s :

3. specific energy ε = 0.06;

4. $\vec{T}$ = 2·126 mN (thruster Phall 1 - UNB);

5. nominal thruster T_o = 1 UA;

6. The time: *time* = 0 corresponds to the date 01/01/2000.

Diverse missions can be implemented with the optimization algorithm used in this work, of which the main ones are: transference with free time (to change to the space vehicle orbit without restrictions to the

necessary time the execution maneuver), Gomes, [12 - 13]; "Rendezvous" (one desires that the space vehicle finds and remains to the side of another space vehicle), Vaz [14]; "Flyby" (desires to intercept another celestial body, however without the objective to remain next); "Swing-By" (to pass by a celestial body to gain or to lose energy, speed and angular moment), etc.

4.1 Mission Asteroid 2002TC70

Using the optimization procedure we can find optimal trajectories, with the maximization of the spacecraft final mass (i.é., minimum fuel consumption). These trajectories depend on the mission objectives, for example, the performance depends on the mission time length. It is possible to reduce the time with some more spend of propellant.

Table 1 – Keplerian Elements

Name	2002TC70
Epoch	54200
a	1.369831
e	0.19691574
i	2.13932
Ω	161.89427
ω	134.84892
M	351.6336031
r_a	1.10009
r_p	1.639572

Were considered similar missions (Direct, EGA, EMGA) for the asteroid 200TC70 and analyzed the parameters considered important in this project, such as semi-major axis (a), eccentricity (e) switching function (Sf), hamiltonian, energy the orbit, the initial mass less consumption of propellant (Δm) (parameter optimization), ΔV electric (ΔVel) Important dates in the mission (departure, arrival and flyby) (Santos, 2009) [15].

4.1.1 - Simulation without flyby

In this section is showed direct orbits with the thruster Phall 1. In Figure 1 is observed trajectory that leaves the Earth and arrives at the asteroid. The mission duration time (ΔT) is approximately 748 days and the vehicle final mass (Δm) is 1673,7 kg.
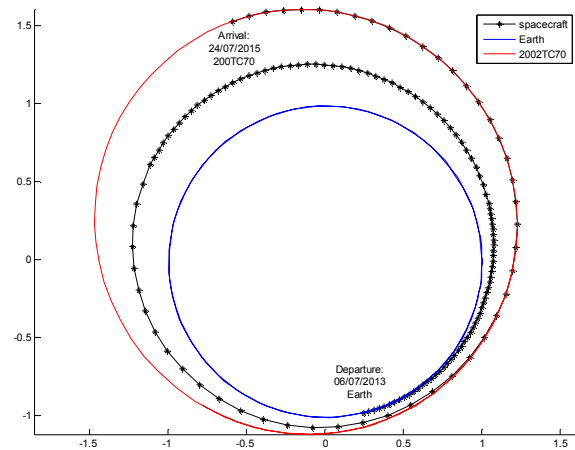

Figure 1 - Trajectory direct (without flyby) to the asteroid 2002TC70, using Phall 1.

4.1.2 – EGA (Earth) mission

In this section, is viewed the orbit transfer leaving Earth, making EGA (flyby in Earth) approximately one year and 3 months after, and, arriving at asteroid 2002TC70 with the total time of the mission (ΔT) of 845.23 days after the departure and the vehicle mass (Δm) is of 1875 kg, showed in Figure 2 and Figure 4.
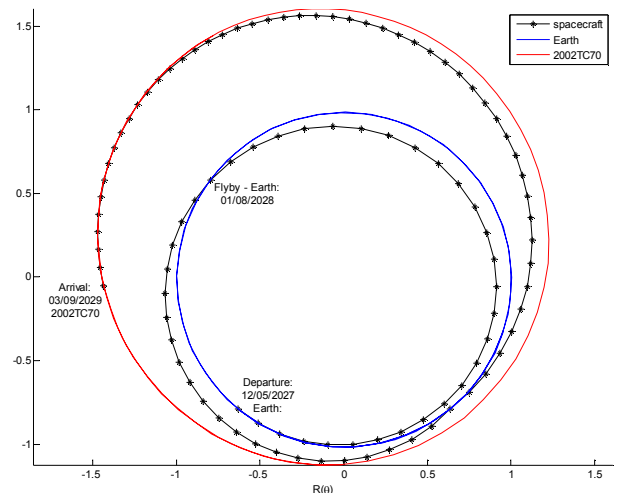

Figure 2 - Trajectory leaving the Earth and arriving to the asteroid, using EGA maneuvers, with Phall 1.

4.1.3 EMGA (Earth – Mars) mission

It resulted better in comparison to the results gotten with PPS 1350 (Table), therefore, Phall 1 possess a bigger specific thrust and the thruster (t) is bigger in magnitude.
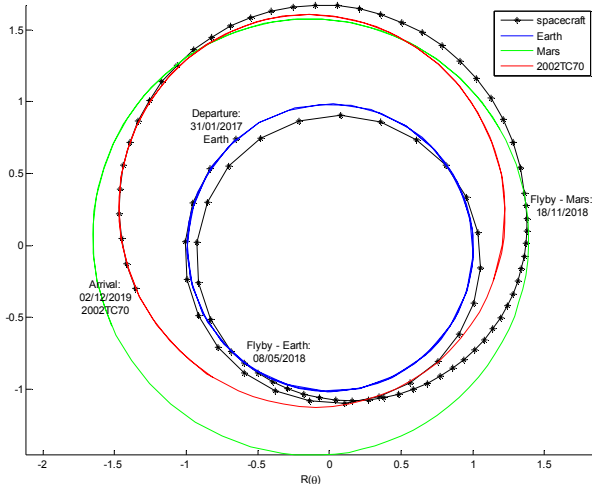

Figure 3 - Trajectory leaving the Earth and arriving to the asteroid, using EMGA maneuvers, with Phall 1.

The Figure 4 exhibit a comparison (asteroid 200TC70 using 2 thrusters: PPS 1350 and Phall) of time and final mass of the vehicle with the use of the optimized maneuver without flyby e with flyby at the Earth (Figures 1, 2), Earth and Mars (Figure 3 and Figure 4);

also compares, the ΔV electric gotten with the maneuver and the related ideal or possible dates for the mission.

Simulations with maneuvers for asteroid had been carried through 1989UQ (Santos, 2009) [15], also using EVGA (first flyby in the Earth and second flyby in Venus) maneuvers, and the results obtained were satisfactory with the use of solar electric energy.

From numerical simulation of spacecraft orbit transfer it is possible to conclude that with higher Phall thrust will be possible to decrease maneuvering time.

Is observed that the direct mission (without flyby) using Phall 1, has the shorter duration in days when compared with missions that involve gravitational maneuvers, however, the propellant consumption is greater in relation the other missions (EGA, EMGA, EVGA). The simulations showed that the use of gravitational maneuvers considerably reduces the consumption of the propellant, when the time are not the main parameter for optimization of the mission. In the comparison must be taken into consideration that Phall 1 has specific impulse (I_s) and thruster (T) with the magnitude higher that PPS 1350. Is noted that the magnitude of the variation of velocity (ΔV_{el} e V_∞) in the direct simulation (without flyby) is greater, because the actions of propellant electric is more effective however the operational cost is higher (measured in Δm).

Asteroid 2002TC70									
PPS 1350 (I_s= 1550s) (2 x 70mN)					Hall (UNB) (Is= 1607s) (2 x 126mN)				
Duration (days)	*Δm (% m_0)*	*Date*	ΔV_{el} (Electric) (km/s)	V_∞ (km/s)	*Duration (days)*	*Δm (% m_0)*	*Date*	ΔV_{el} (Electric) (km/s)	V_∞ (km/s)
Flyby: 0					*Flyby: 0*				
1052.85	77.68%	Departure: 14/07/2013 Arrival: 01/06/2016	3.83917757	1.25579628	748.24	78.46%	Departure: 06/07/2013 Arrival: 24/07/2015	3.820898907	1.117814179
Flyby: 1 – Earth Gravity Assisted (EGA)					*Flyby: 1 – Earth Gravity Assisted (EGA)*				
883.40	86.84%	Departure: 13/05/2016 *Flyby* - Earth: 30/07/2017 Arrival: 13/10/2018	2.14480657	0.8376562	845.23	87.89%	Departure: 12/05/2027 *Flyby* - Earth: 01/08/2028 Arrival: 03/09/2029	2.034274163	0.870011798
Flyby: 2 - Earth Mars Gravity Assisted (EMGA)					*Flyby:* 2 - Earth Mars Gravity Assisted (EMGA)				
1081.15	87.39%	Departure: 11/02/2017 *Flyby* - Earth: 11/05/2018 *Flyby* - Mars: 17/11/2018 Arrival: 28/01/2020	2.04883921	0.58081769	1035.51	88.86%	Departure: 31/01/2017 *Flyby* - Earth: 08/05/2018 *Flyby* - Mars: 18/11/2018 Arrival: 02/12/2019	1.86129921	0.647333385

Figure 4 - *Comparative table of the use of the flyby for asteroids 2002TC70 using two Thrusters.*

5 Conclusion

The search for the best initial parameters for a mission is facilitated if the transfer orbit with free time is optimized first. The ideal asteroids for EGA missions should possess low orbit energy, perihelion close to 1 UA, low inclination per EGA.

Indirect optimization methods based on optimal control theory supply accurate solutions. The use of Gravity Assisted Maneuver (EGA, EMGA or EVGA) in this mission reduces the fuel consumption and the time of the maneuver, demonstrating that this important formulation is viable and useful.

Orbits with Phall 1 had been analyzed using gravity assisted maneuvers and verified resulted optimistical for the implantation of probes using this technology, also being able to use this formularization in the future missions that use launch vehicle that is in development/improvement (VLS-2, Brazil), which can inject in LEO (low earth orbit) a satellite medium sized, thereafter, use the solar electric propulsion (SEP) or nuclear (NEP) to dislocate the vehicle for desired orbits, maximizing them with the maneuver that use assisted gravity.

The present analysis favor a guess at the tentative solution as the Earth's positions as departure and flyby are a priori known. The ideal asteroid has perihelion radius which is close to 1AU, a low-energy orbit and low inclination with relation to the ecliptical axis.

The performance parameters of Phall are competitive with known electromagnet Hall thrusters found on the literature.

The fuel consumption for a mission with multiples flyby's follows the criterion of the asteroid orbit.

Acknowledgment

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