ORBITAL CHARACTERISTICS DUE TO THE THREE DIMENSIONAL SWING-BY IN THE SUN-JUPITER SYSTEM

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Abstract:-This paper presents an analytical and numerical study about orbital characteristics of trajectories due to a three dimensional swing-by maneuver between a planet and a particle. The model used has the same hypothesis of the circular restricted three-body problem. It is assumed that the forces are given by two main bodies that are in circular orbits around their center of mass and that the particles are moving under the gravitational attraction of these two primaries. This method has been under study for a long time by several authors, most of them used the dynamical system given by the "patched-conics". The technique shown here generates accurate solutions for interplanetary transfer description. Two solutions are considered for the Swing-By (clock-wise and counter-clock-wise orbit). The goal is to study the orbital change due to the variation of inclination, longitude of the ascending node and argument of periapsis, considering two angles of approaches. Finally, numerical simulations are performed using the Sun-Jupiter system to determine the evolution of the cloud of particles during the maneuver and to map regions for optimal maneuvers.

Key-words:- orbital maneuver, three dimensional swing-by, celestial mechanics, orbital dynamics

1 Introduction

It is known that the study to optimize trajectories for space missions has increased in recent years, particularly when concerning minimizing the fuel consumption of spacecrafts. On the other hand, some strategies used during the insertion phases of the missions are quite complex. The procedure to minimize the number of activities required by the spacecraft during the mission is accomplished by the balance against propellant cost associated with delaying corrections until after orbit insertion.

In the neighborhood of a planet, a spacecraft in a orbit around the Sun experiences perturbations which depend on the relative velocity between the spacecraft and the planet and the distance separating the two of them at the point of the closest approach. If only the gravitational field of the planet affects the motion of the spacecraft, the vehicle would make its approach along a given trajectory. A method frequently used in the literature to study the gravitational effect of the planets to change the trajectories of bodies is called patched-conic. It involves partitioning the overall trajectory into several two-body problems. The technique of transfers assumes that the sphere of influence of a planet has an infinite radius when observed from the planet, and has zero radius when observed from the Sun. Trajectories within the sphere of influence are studied by the model given by the two body problem, with the planet as the primary attracting body. In other words, only one celestial body influence the trajectory of the spacecraft for a given time. The standard maneuver uses a close approach with a celestial body to modify the velocity, orbital elements, energy and angular momentum of the spacecraft or several particles.

Although the most usual approach to study this problem is to divide the problem in three phases dominated by the "two-body" celestial mechanics, other models are also used to study this problem, like the circular restricted three-body problem [1], [2], and [6] and the elliptic restricted three-body problem [10].

The literature shows several applications of the swing-by technique. Some examples are: a mission to study the Earth's geomagnetic tail [3]; a swing-by in three dimensions, including the effects in the inclination [5]; a study of the effects of the atmosphere in a swing-by trajectory [7]; a swing-by with the Moon [8]; a swing-by maneuver applying an impulse during the passage by the periapsis [9]; a swing-by in Venus to reach Mars [13]; a mission to Neptune using swing-bys to gain energy to accomplish the mission [15], etc.. On the other hand, a numerical study with more detail of the planar restricted three-body can be found in [17] and [18].

This paper will use the Swing-By maneuver (gravity-assist) to analyze missions involving Jupiter and a cloud of particles. With the use of analytical equations for the variations of velocity, energy, angular momentum and inclination presented in reference [5], the study will be to extend those equations to determine the variation of longitude of the ascending node and variation of the argument of periapsis considering two close approaches. The maneuver uses a three dimensional swing-by with a celestial body (M_2) to modify the energy, angular momentum, velocity and orbital elements of the particles with respect to the Sun (M_1). The goal is to find an economical strategy to change the inclination, longitude of the ascending node and argument of periapsis of the orbit of the particles by using a close approach with Jupiter. Therefore, it will be possible to accompany the evolution of the particles and to map regions for optimal maneuvers.

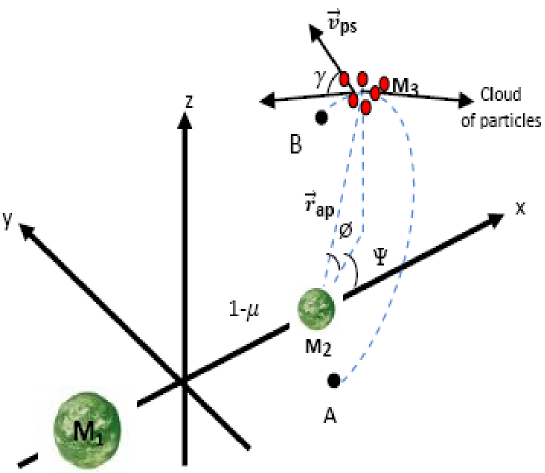

Fig. 1-Three dimensional swing-by for a cloud of particles.

2 The Three-dimensional Circular Restricted Problem and Swing-by: Analytical equations

In this study, the equations of motion for the particle are assumed to be the ones valid for three-dimensional restricted circular three body problem. The standard dimensionless canonical system of units will be used, which implies that:

i. the mean angular velocity of the motion of M_1 and M_2 is assumed to be one;
ii. the unit of distance is the distance between M_1 and M_2;
iii. the mass of the smaller primary (M_2) is given by $\mu = m_2/(m_1 + m_2)$. The m_1 and m_2 are real masses of M_1 and M_2, respectively;
iv. The mass of M_1 is 1-μ;
v. The unit of time is defined such that the period of motion of the two primaries is 2π and the gravitational constant is one.

The rotating system of reference will be used, which has the origin at the center of mass of M_1 and M_2. The vertical axis (y) is perpendicular to the axis (x), which rotates with a variable angular velocity in a such way that M_1 and M_2 are always on this axis (Fig. 1). In this system, the position of M_1 and M_2 are: $y_1 = y_2 = 0$, $x_1 = -\mu$, $x_2 = 1-\mu$, $y_1 = y_2 = 0$. The equations of motion for this system are [14]:

$$\begin{aligned} \ddot{x} - 2\dot{y} &= x - (1-\mu)\frac{x+\mu}{r_1^3} - \mu\frac{x-1+\mu}{r_2^3} \\ \ddot{y} - 2\dot{x} &= y - (1-\mu)\frac{y}{r_1^3} - \mu\frac{y}{r_2^3} \\ \ddot{z} &= (1-\mu)\frac{z}{r_1^3} - \mu\frac{z}{r_2^3} \end{aligned} \tag{1}$$

where r_1 and r_2 are the distances from M_1 and M_2.

To start the description of the mathematic model used in this paper, the initial conditions with respect to M_2 at the periapsis of this trajectory are calculated. The initial position and the initial velocity of these points can seen in Fig. 1 and they are given by [5]:

$$\begin{aligned} x_i &= r_{ap}\cos\emptyset\cos\Psi \\ y_i &= r_{ap}\cos\emptyset\sin\Psi \\ y_i &= r_{ap}\sin\emptyset \end{aligned} \tag{2}$$

$$\begin{aligned} v_{xi} &= -V_p\sin\gamma\sin\emptyset\cos\Psi - V_p\cos\gamma\sin\Psi \\ v_{yi} &= -V_p\sin\gamma\sin\emptyset\sin\Psi + V_p\cos\gamma\cos\Psi \\ v_{zi} &= V_p\cos\emptyset\sin\Psi \end{aligned} \tag{3}$$

Where $\vec{r}_{ap}$ is the distance from the spacecraft to the center of M_2, V_p is the velocity of M_3 with respect to M_2 and ψ is the angle of approach.

When the spacecraft has a close approach with M_2, it is assumed that the two-body problem is valid and the whole maneuver takes place in the plane defined by the vectors $\vec{r}_{ap}$ and $\vec{v}_p$. So, it is possible to determine the velocity of the particle with respect to M_1 in the moment of the crossing

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with the planet's orbit and the true anomaly of that point. The velocity and true anomaly are:

$$|V_i| = \sqrt{\mu_s\left(\frac{2}{r_{sp}} - \frac{1}{a}\right)} \tag{4}$$

$$\theta = \mathrm{Cos}^{-1}\left[\frac{1}{e}\left(\frac{a(1-e^2)}{r_{sp}} - 1\right)\right] \tag{5}$$

The parameter r_{sp} is the distance between M_1 and M_2, *a* is the semi-major axis and *e* is eccentricity of the orbit. The velocity and orbital elements of M_3 are changed when it has a close approach with M_2. So, Eq. (5) given us two solutions (θ_A and θ_B), but in this paper only the positive angle will be considered. The next procedure is to calculate the angle between the inertial velocity of the particle and the velocity of the planet:

$$\gamma = \tan^{-1}\left[\left(\frac{e \,\mathrm{Sin}\,\theta}{1 + e\,\mathrm{Cos}\,\theta}\right)\right] \tag{6}$$

The next step is to calculate the magnitude of M_1 velocity with respect to M_2 in the moment that the approach starts:

$$V_\infty = \sqrt{V_i^2 + V_2^2 - 2V_iV_2\,\mathrm{Cos}\,\gamma} \tag{7}$$

where V_i is the velocity of the particle with respect to M_1 and $\vec{V}_2$ is the velocity of M_3 with respect to M_1, with

$$\vec{V}_2 = (0, V_2, 0) \tag{8}$$

In canonical units V_2 is one due the fact the distance between M_2 and M_1 is one. This study considers two solutions assuming a close approach behind the planet (rotation of the velocity vector in counter-clock-wise sense- ψ_1) and close approach in front of planet (clock-wise sense- ψ_2) for the spacecraft around the Sun (Fig. 2). These two values are obtained from:

$$\psi_1 = 180^0 + \beta + \delta$$
$$\psi_2 = 360^0 + \beta - \delta \tag{9}$$

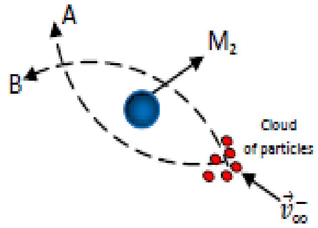

Fig. 2-Possible rotation of the velocity vector.

where [1]

$$\beta = \mathrm{Cos}^{-1}\left[-\frac{V_i^2 - V_2^2 - V_\infty^{-2}}{2\,V_2\,V_\infty^-}\right]$$

$$\delta = \mathrm{Sin}^{-1}\left[\frac{1}{1 + \frac{r_p V_\infty^2}{\mu_p}}\right] \tag{10}$$

μ_p is gravitational constant of the planet, δ is half of the angle of curvature, $\vec{V}_\infty^-$ and $\vec{V}_\infty^+$ are the velocities of M_3 with respect to M_2, before and after the maneuver, in the referential frame considering the three dimensional frame [5]:

$$\vec{V}_\infty^- = V_\infty\,\mathrm{Sin}\delta\,(Cos\,\psi\;Cos\,\phi, Sin\,\psi\;Cos\,\phi, S$$
$$-\,\mathrm{Cos}\,\gamma\;\;\mathrm{Sin}\,\psi, \mathrm{Sin}\,\gamma\;\;\mathrm{Sin}\,\phi\;S$$

$$\vec{V}_\infty^+ = -V_\infty\,\mathrm{Sin}\delta\,(Cos\,\psi\;Cos\,\phi, Sin\,\psi\;Cos\,\phi,$$
$$-\,\mathrm{Cos}\,\gamma\;\;\mathrm{Sin}\,\psi, \mathrm{Sin}\,\gamma\;\;\mathrm{Sin}\,\phi\;S \tag{11}$$

From Eq.(8) and Eq.(11) we can determine $\vec{V}_i$ and $\vec{V}_o$, that are the velocities of M_3 with respect to the inertial frame before and after the swing-by, respectively:

$$\vec{V}_i = \vec{V}_\infty^- + \vec{V}_2$$
$$\vec{V}_o = \vec{V}_\infty^+ + \vec{V}_2 \tag{12}$$

From those equations, it is possible to obtain expressions for the variations in velocity, energy, and angular momentum for the three dimensional swing-by, respectively [5]:

$$\Delta V = |\overline{\Delta V}| = |\vec{V}_o - \vec{V}_i| = 2\,V_\infty \mathrm{Sin}\delta$$

$$\Delta E = -2\,V_2\,V_\infty \mathrm{Sin}\delta\;\mathrm{Sin}\psi\;Cos\,\phi \tag{13}$$

$$\Delta C = \left|\vec{C}_o - \vec{C}_i\right| = 2dV_\infty \mathrm{Sin}\delta\sqrt{Cos^2\phi\, Sin^2\psi +}$$

where $\vec{R} = (d,0,0)$ is the position vector of M_2 and $\vec{C}_i$ and $\vec{C}_o$ are the angular momentum vector of M_3 with respect to the referential frame before and after the swing-by for each particle. From Eq. (12) the angular momentum is assumed be:

$$\begin{aligned} \vec{C}_i &= \vec{R} \times \vec{V}_i = (C_{ix}, C_{iy}, C_{iz}) \\ \vec{C}_o &= \vec{R} \times \vec{V}_o = (C_{ox}, C_{oy}, C_{oz}) \end{aligned} \qquad (14)$$

3 Analytical Model for the Orbital Elements in the Three Dimensional Swing-by

Assuming that r_p is a parameter, it is possible to determine the variations of the orbital elements after the swing-by as a function of the orbital elements before the maneuver and it is variations. The analytical form to study the orbital elements in the three-dimensional swing-by after the close approach for each particle is assumed:

$$\begin{aligned} a &= \frac{(1-\mu)}{2(1-\mu) - V_o^2} \\ e &= \sqrt{1 - \frac{C_o^2}{a(1-\mu)}} \end{aligned} \qquad (15)$$

Where V_0 is the velocity of M_3 with respect to the inertial frame after the swing-by; C_0 is the angular momentum of M_3 with respect to the referential frame after the swing-by for each particle; a and e are the semi-major axis and eccentricity at the encounter, respectively. With those equations available and from Eq. (13) and Eq.(14), the results for inclinations are:

$$Cos\, i_k = \frac{C_{zk}}{\left|\vec{C}_k\right|} \qquad (16)$$

The index $k = i$ and $k = 0$ are related to before and after the maneuver, respectively. So, i_i and i_o are the inclinations before and after the close approach for each particle.

The analytical expressions to determine the longitude of the ascending node before and after the swing-by, respectively, is:

$$\Omega_k = \mathrm{Cos}^{-1}\left(\frac{-C_{yk}}{\left|\vec{C}_k\right|}\right) \qquad (17)$$

where, from Eq.(14), we can determine

$$\Omega_i = \mathrm{Cos}^{-1}\left[\frac{V_\infty D_i}{r_{ap}\sqrt{A_i^2 + B_i^2}}\right]_{\#} \qquad (18)$$

with

$D_i =$
$r_{ap} Cos\,\gamma\, Cos\,\delta\, Sin\,\psi\, Sin\,\phi +$
$r_{ap}\, Cos\,\psi\, Cos\,\delta\, Sin\,\gamma + d\, Cos\,\phi\, Cos\,\delta\, Sin\,\gamma +$
$d\, Sin\,\phi\, Sin\,\delta$

;

$A_i = (V_2 + V_\infty Cos\,\phi\, Cos\,\gamma\, Cos\,\delta) Sin\,\phi -$
$V_\infty Cos\,\delta\, Sin\,\psi\, Sin\,\gamma$

;

$B_i - V_\infty(r_{ap} Cos\,\gamma\, Cos\,\delta\, Sin\,\psi\, Sin\,\phi +$
$r_p Cos\,\psi\, Cos\,\delta\, Sin\,\gamma + d\, Cos\phi\, Cos\,\delta\, Sin\,\gamma +$
$d\, Sin\,\phi\, Sin\,\delta)$

;

and after the encounter is

$$\Omega_o = \mathrm{Cos}^{-1}\left[\frac{V_\infty D_o}{r_{ap}\sqrt{A_o^2 + B_o^2}}\right] \qquad (19)$$

with

$D_o - V_\infty(r_{ap} Cos\,\gamma\, Cos\,\delta\, Sin\,\psi\, Sin\,\phi +$
$r_{ap} Cos\,\psi\, Cos\,\delta\, Sin\,\gamma + d\, Cos\,\phi\, Cos\,\delta\, Sin\,\gamma -$
$d\, Sin\,\phi\, Sin\,\delta)$

;

$A_o = A_i;$

$B_o = V_\infty(r_{ap} Cos\,\gamma\, Cos\,\delta\, Sin\,\psi\, Sin\,\phi +$
$r_{ap} Cos\,\psi\, Cos\,\delta\, Sin\,\gamma + d\, Cos\,\phi\, Cos\,\delta\, Sin\,\gamma -$
$d\, Sin\,\phi\, Sin\,\delta)$

;

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In Eq.(19), the parameter d is the distance from M_1 to M_2, that, in the canonical units, is one. Next, it is calculate the argument of periapsis (ω) before and after the swing-by, that in astrodynamics can be calculate as follows:

$$\omega_k = Cos^{-1}\left(\frac{\vec{n}_k \cdot \vec{e}_k}{|\vec{n}_k||\vec{e}_k|}\right) \tag{20}$$

In Eq. (20), we have that n_k is the vector pointing towards the ascending node (i.e. the z-component of n is zero), with

$$n_k = \sqrt{C_{xk}^2 + C_{yk}^2} \tag{21}$$

Already e_k is the eccentricity vector (the vector pointing towards the periapsis), that we can determine through the expression [16]:

$$\vec{e}_k = e_{xk}\hat{\imath} + e_{yk}\hat{\jmath} + e_{kk}\hat{k} \tag{22}$$

with

$$e_{xk} = \frac{1}{\mu}(Ax_k + Bv_{xk}),$$

$$e_{yk} = \frac{1}{\mu}(Ay_k + Bv_{yk}),$$

$$e_{zk} = \frac{1}{\mu}(Az_k + Bv_{zk}),$$

And $A = v_p^2 - \mu/(r_{ap})$, $B = -r_{ap} \cdot v_p$

The term v_p represents the velocity of the particle, r_{ap} is the distance between M_2 and M_3 for each particle and μ is the mass of the smaller primary (M_2), that is is given by $\mu = m_2/(m_1 + m_2)$.

If $e_k < 0$ then the argument of periapsis is obtained by:

$$\omega_k = 2\pi - Cos^{-1}\left(\frac{\vec{n}_k \cdot \vec{e}_k}{|\vec{n}_k||\vec{e}_k|}\right).$$

In the case of circular orbits it is often assumed that the periapsis is placed at the ascending node and therefore $\omega = 0$.

So, from equations (20)-(22) and Eq.(14), we can determine the argument of periapsis before the encounter:

$$\omega_i = Cos^{-1}\left(\frac{A_{\omega i}}{B_{\omega i}}\right) \tag{23}$$

with

$$A_{\omega i} = r_{ap}\, Cos\,\emptyset \left(d^2 + r_{ap}{}^2 + 2\, d\, r_{ap}\, Cos\,\psi\, Cos\emptyset\right)^{\frac{1}{4}} \{ r_{ap}\, Sin\,\psi\, Sin\,\emptyset\, (V_2 - V_\infty\, Cos\,\psi\, Cos\,\gamma\, Cos\,\delta + V_\infty Cos\,\emptyset\, Cos\,\gamma\, Cos\,\delta) - V_\infty [(r_p + d\, Cos\,\psi\, Cos\,\emptyset) Cos\,\delta\, Sin\gamma + d\, Cos\,\psi\, Sin\,\emptyset\, Sin\delta] \}$$

;

$$B_{\omega i} = \left[(Sin\,\emptyset\, (V_2 + V_\infty Cos\,\emptyset\, Cos\,\gamma\, Cos\,\delta) - V_\infty\, Cos\,\delta\, Sin\,\psi\, Sin\,\gamma)^2 + V_\infty{}^2 (r_{ap}\, Cos\,\gamma\, Cos\,\delta\, Sin\,\psi\, Sin\,\emptyset + r_{ap}\, Cos\,\psi\, Cos\,\delta\, Sin\,\gamma + d\, Cos\,\emptyset\, Cos\,\delta\, Sin\,\gamma + d\, Sin\,\emptyset\, Sin\,\delta)^2 \right]^{1/2}$$

;

After the swing-by, we have

$$\omega_o = arc\, Cos\left(\frac{A_{\omega o}}{B_{\omega o}}\right) \tag{24}$$

With

$$A_{\omega o} = r_{ap}\, Cos\,\emptyset \left(d^2 + r_{ap}{}^2 + 2\, d\, r_{ap}\, Cos\,\psi\, Cos\emptyset\right)^{\frac{1}{4}} \{ r_{ap}\, Sin\,\psi\, Sin\,\emptyset\, (V_2 - V_\infty\, Cos\,\psi\, Cos\,\gamma\, Cos\,\delta + V_\infty Cos\,\emptyset\, Cos\,\gamma\, Cos\,\delta) - V_\infty [(r_{ap} + d\, Cos\,\psi\, Cos\,\emptyset) Cos\,\delta\, Sin\gamma - d\, Cos\,\psi\, Sin\,\emptyset\, Sin\delta] \}$$

;

and from Eq. (25), we have $B_{\omega o} = B_{\omega i}$

In the case of circular orbits it is often assumed that the periapsis is placed at the ascending node and therefore $\omega = 0$.

4 Singularities and Regions out-of-plane

In this section, the study of the swing-by maneuver is extended to consider non-zero values for the out-of-plane component for analyses of the variation of the inclination, longitude of the ascending node and argument of periapsis. The goal is to verify some characteristic regions due to the swing-by maneuver. This study will be important to analyze the evolution of the particles and to determine the initial conditions where an optimal maneuver is performed.

Fig. 3 shows that there are several null values for the variations of the inclination and also

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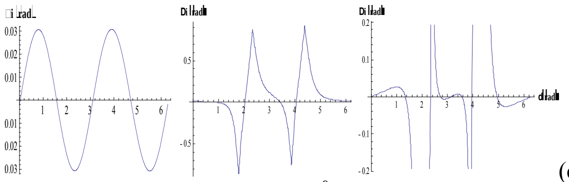
maximum inclinations. However, in Fig.3-c, considering the half angle of curvature (δ), it is possible to see three ranges where the singularities occur. The maximum amplitude for inclination is 0.9, when studying this problem as a function of the close approach angle (Ψ). An overview of the regions of variations for inclinations can be seen with detail in Fig. 4, where it is possible to choose an inclination that minimizes the variations. The maximums and minimums of those oscillations are also dependent on the initial conditions. The red and blue indication (Fig.4) shows the maximum and minimum amplitude of the inclination, respectively.



(a) $\gamma=\Psi=\delta=15^0$, $\beta=30^0$ (b)$\gamma=$ ø$=\delta=15^0$, $\beta=30^0$ (c)$\gamma=$ ø $=\Psi=15^0$, $\beta=30^0$

Fig. 3- Variation of inclination as function ø, ψ and δ, respectively $v_2 = 1$; $r_{ap}= 0.0045848$, d =1, $v_\infty = 2$, $v_p = 3.4$, $\mu = 9.5507\text{x}10^{-4}$.

The amplitude of variation of the longitude of the ascending node as a function of φ, ψ and δ can be seen in Fig. 5 and Fig. 6. Analyzing Fig. 5 we can see that there are no singularities due to the swing-by maneuver for the longitude of the ascending node and that $\Delta\Omega$ has minimum variations as a function of the close approach (Ψ).

In Fig. 5-a and Fig. 5-c it is possible to see a symmetric and similar range (0 rad $\leq \delta \leq$ 0.85 rad and 2.25 rad $\leq \delta \leq$ 4.25 rad) when $\Delta\Omega = 0$. The minimal variations (0.0035 rad or 0.2 deg) occur for $\Delta\Omega$ as a function of the close approach angle (Ψ). Already, it is possible to see the great influence of the deflection angle (δ).

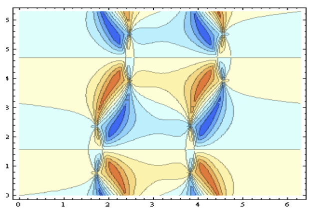
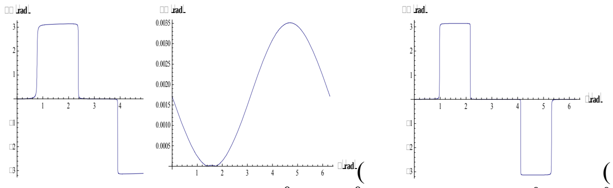


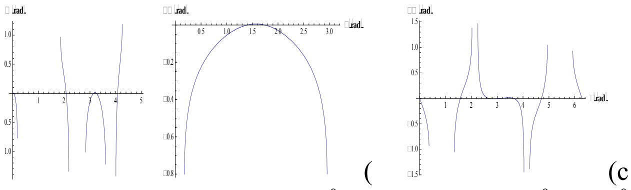
Fig.4- 2D view of variation of the inclination for Ø vs Ψ for $\gamma = \delta = 15^0$, $\beta = 30^0$, $v_2 = 1$; $r_{ap}= 0.0045848$, d =1, $v_\infty = 2$, $v_p = 3.4$, $\mu = 9.5507\text{x}10^{-4}$



(a)$\gamma=\Psi=\delta=15^0$, $\beta=30^0$ (b)$\gamma=$ ø$=\delta=15^0$, $\beta=30^0$ (c)$\gamma=$ ø$=\Psi=15^0$, $\beta=30^0$

Fig.5-Amplitude of variation of Longitude of the ascending node as function ø, ψ and δ, respectively, for $v_2 = 1$; $r_{ap}= 0.0045848$, d =1, $v_\infty = 2$, $v_p = 3.4$, $\mu = 9.5507\text{x}10^{-4}$.

Fig. 6-a and Fig. 6-c shows several singular values for out-of-plane in the variation of the argument of periapsis. The zero divisor occur in $\Delta\omega$ when the close approach angle ($\Psi=15^0$) is fixed and ø and δ are varying. Several initial conditions that can be considered to obtain an optimal mission are presented in Fig. 6 and Fig. 7. The white regions in Fig.7 are singularities regions for the argument of periapsis.



(a)$\gamma=\Psi=\delta=15^0$, $\beta=30^0$ (b)$\gamma=$ ø$=\delta=15^0$, $\beta=30^0$ (c)$\gamma=$ ø$=\Psi=15^0$, $\beta=30^0$

Fig.6-Amplitude of variation of the argument of periapsis as function on ø, ψ and δ, respectively for $v_2 = 1$; $r_{ap}= 0.0045848$, d =1, $v_\infty = 2$, $v_p = 3.4$, $\mu = 9.5507\text{x}10^{-4}$.

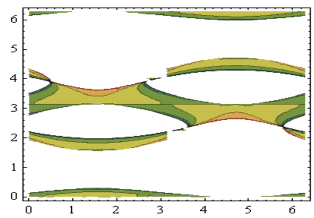


Fig.7- 2D view of variation of the Argument of periapsis as function of ø and ψ for $\gamma = \delta = 15^0$, $\beta=30^0$, $v_2 = 1$; $r_{ap}= 0.0045848$, d =1, $v_\infty = 2$, $v_p = 3.4$, $\mu = 9.5507\text{x}10^{-4}$.

5 The Orbital Change of Particles: Numerical Analyses of the Problem

Based in the initial conditions observed in section 4, some simulations will be performed to analyze the orbital variation of the cloud of particles subject to a close approach with Jupiter. It is assumed that the particles are in orbit around the Sun with given semi-major axis and eccentricity and the periapsis distance (r_p) and apoapsis distance (r_a) are assumed to be known.

The simulations are performed with the following characteristics: the Sun (or the other perturbations) does not affect the motion of the particles; the orbital elements will be analyzed after and before the maneuver for some values of the semi-major axis and eccentricity considering a cloud of particles. The solution 1 will be performed for the first maneuver behind the planet and solution 2 for the first maneuver performed in front of the planet. The initial conditions are: r_{ap} = 50 Rj (Jupiter radius), v_p = 4, both in canonical units and with initial inclination ø=30^0.

Fig. 8 shows the variation of the angle of close approach as a function of the semi-major axis. In this figure we can see the range of variation for both cases and that it does not have a dispersion of the particles through the semi-major axis. These results are important to choose an angle of approach that makes minimal changes during the maneuver. The influence of the angle Ψ in the variations of the orbital elements was analyzed.

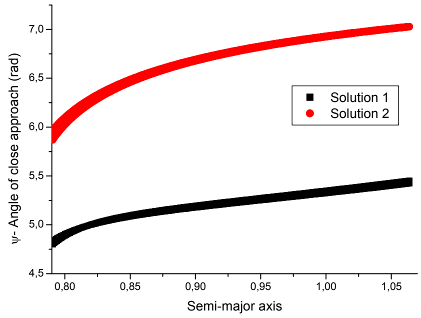


Fig.8- Angle of approach as a function of the semi-major axis

From the analyses of the areas, it is possible to see a difference in the behavior between both solutions (Figs. (9)-(14)). In Fig. 9 the amplitude of the variation in the eccentricity doesn´t have a similar behavior, showing a decreasing for solution 2, reaching the minimum value and then a dispersion of the particles in the end of the trajectory.

The variation of inclination of the orbit after the swing-by has a strong influence from the initial inclination and semi-major (Fig. 10). Solution 1 shows that the particles are concentrated at the end of the trajectory and reaches a minimum variation of the inclination. Solution 2 shows a large variation in the inclination and a similar dispersion of the particles, as seen in Fig. 9, but have no relationship between the eccentricity and inclination. This is possible due to the energy gain that occurs when the particles pass behind Jupiter.

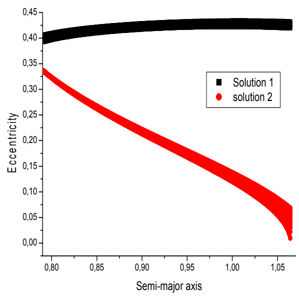


Fig. 9- Eccentricity vs semi-major axis.

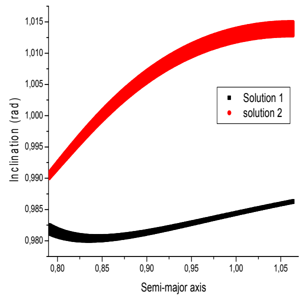


Fig. 10- Inclination vs semi-major axis.

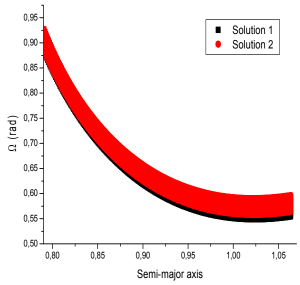


Fig. 11- Longitude of the ascending node vs semi-major axis.

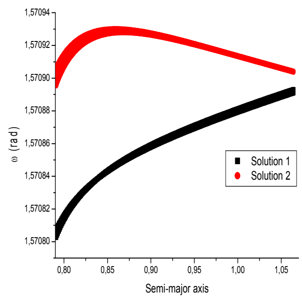


Fig. 12- Argument of periapsi vs semi-major axis

We can see in Fig. 11 and Fig. 14 a minimum difference between solution 1 and solution 2 for the variation of the longitude of the ascending node and the angular momentum, respectively. Furthermore, in Fig. 11, we can see a large decrease in the longitude during the maneuver. Those preliminary results indicate that further studies have to be performed to obtain more indications of this phenomenon, showing the relationship between these two variables.

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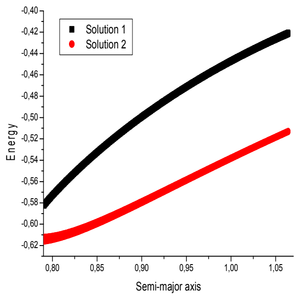

Fig. 13- Energy vs semi-major axis.

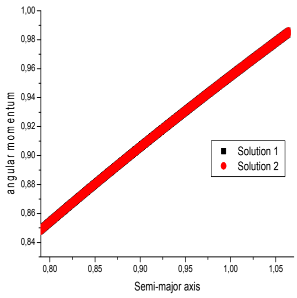

Fig. 14-Angular momentum vs semi-major axis.

It is visible that there is a difference in the variation of energy observed in Fig. 13. This result was expected because of the effects of the swing-by increases when the particles is passing closer to Jupiter. The two solutions considered, depending on the geometry of the encounter, has different behaviors. The cases where occurs the increase in energy generates parabolic and hyperbolic orbits after the passage.

6 Conclusion

The effects of the close approach in the longitude of the ascending node, argument of periapsis and inclination of the spacecraft are studied and the results show several particularities. Depending on the angle of approach, it has little influence in the variation of the inclination in the cases considered. There are no singularities due to the swing-by maneuver for the longitude of the ascending node and $\Delta\Omega$ has minimum variation as a function of the angle of approach (Ψ). The longitude of the ascending node and the angular momentum showed a minimum difference between solution 1 and solution 2, but a large dispersion of the particles. The eccentricity of the particles showed a large variation due to the increase of the semi-major axis. The argument of the periapsis has a large variation in both solutions, but the maximum variation occur when the particles are closer to Jupiter.
The study and simulations showed the importance of changing the values for the variation in the semi-major axis and eccentricity, as described in the plots. In this way, this research can be used by mission designers to obtain specific mission goals.

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