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ADAPTIVE NODE-TO-NODE PINNING CONTROL OF COMPLEX NETWORKS

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1. ABSTRACT

A great variety of complex systems can be modeled as networks of interconnected dynamical agents (nodes of the network), which are nonlinear dynamical system, communicating via a communication protocol defined on the network edges that can themselves form complex topologies. Researchers in different areas of applied science and engineering have been addressing the problem of studying how the network topology and the communication protocols between agents determine the way the complex network perform a certain function. Examples include rendezvous and flocking problems in robotics [1], synchronization of sensor networks [2], consensus and multi-agent coordination problems in control theory [3, 4], the emergence of coordinated motion in animal behavior and systems biology (see for instance [5, 6]).

Coordinated motion, consensus, and synchronization are the keywords of the examples cited above; that is because synchronization is a typical collective behavior in nature and technology. Since the pioneering work of Pecora and Carroll [7], chaos control and synchronization have received a great deal of attention; in particular, much research attention has been focused on synchronization [7, 8] and consensus of complex networks [4, 9]. The idea consists in finding strategies to regulate the behavior of large ensembles of interacting agents so as to make all systems in the network evolve towards the same asymptotic evolution which, in general, is unknown *a priori* [10, 11]. The first attempts to solve this problem assumed diffusive coupling between identical nonlinear systems with each of the nodes in the network taking the form

$$\dot{\mathbf{x}}_i(t) = f(\mathbf{x}_i(t), t) - \sigma \sum_{j\in\mathcal{N}_i} \Gamma(\mathbf{H}(\mathbf{x}_i(t)) - \mathbf{H}(\mathbf{x}_j(t))), \quad (1)$$

with $\mathbf{x}_i(t)$ being the state vector of node i, $f(\mathbf{x}_i(t), t)$ the vector field describing the node dynamics, σ a unique global coupling strength between nodes assumed to be constant and time-invariant, Γ is the inner coupling matrix, and $\mathbf{H}$ is the coupling function that for diffusive coupling is defined as $\mathbf{H}(\mathbf{x}) = \mathbf{x}$. $\mathcal{N}_i$ is the set of neighbors of node i, that is, the set of nodes directly connected to node i. Given the node dynamics, the problem then becomes that of finding, for what range of the values of σ the network synchronizes. Such problem termed as the *synchronizability problem* has been solved mainly by using the so-called Master Stability function approach (firstly introduced in [12]).

However, in the case where the whole network cannot synchronize by itself, or neither by adjusting its coupling strength, some controllers may be designed and applied to force the network to synchronize. The idea consists in controlling just a fraction of network nodes by adding some local feedback injections to them, which is known as pinning control, firstly discussed in Grigoriev et al. [13], in which pinning control of spatiotemporal chaos in coupled map lattices was presented.

The problem, however is not only determining the coupling strength, the control gain and the form of the control action to be added to the "pinned" nodes but also how many, and what type of nodes need to be selected in order to achieve the control objective with best performance. The problem of determining the number and type of nodes to be pinned, also termed as *pinning controllability*, was discussed in [14] and [15] where sufficient conditions for the asymptotic solution of the desired common solution were also given. The problem of selecting the type of node that should be pinned in order to improve synchronization performance is still an open problem, however, it has been shown by Porfiri et al. [16]that node-to-node pinning strategy maximizes synchronization performance. In node-to-node pinning control just one node is pinned, and at each instant of time multiple of the switching period T, a new node is randomly chosen to be pinned.

Now, consider the controlled network

$$\dot{\mathbf{x}}_i(t) = f(\mathbf{x}_i(t), t) - \sum_{j\in\mathcal{N}_i} \sigma_{ij}\Gamma(\mathbf{H}(\mathbf{x}_i(t)) - \mathbf{H}(\mathbf{x}_j(t)))$$
$$-\delta_i q_i(\mathbf{H}(\mathbf{x}_i(t)) - \mathbf{H}(\mathbf{x_s}(t))), \qquad i = 1, \ldots, N, \quad (2)$$

where the last term is the pinning control term, whith q_i is the control gain of node i; σ_{ij} is the coupling strength between nodes i and j; N_{pin} is the number of pinned nodes; $\mathbf{x_s}$ is the desired synchronous solution to be achieved - $\mathbf{x_s}$ is also a solution of $f(\mathbf{x}(t), t)$ (that we call reference node) being a equilibrium point, a periodic or chaotic orbit.

Suppose the coupling strength and control gain can be adaptively set via adaptation laws given by the equations (3) and (4) respectively, see [17] for details.

$$\dot{\sigma}_{ij}(t) = \alpha \|\mathbf{x}_i(t) - \mathbf{x}_j(t)\|^p, \ \forall (i,j) \in \mathcal{N}, \ 0 < p \leq 2 \quad (3)$$

with $\alpha > 0$, and $\mathcal{N} = \sum_{i=1}^{N} \mathcal{N}_i$.

$$\dot{q}_i(t) = \kappa \|\mathbf{x}_i(t) - \mathbf{x}_s(t)\|^p, \ 0 < p \leq 2 \quad (4)$$

with $\kappa > 0$, where $i = 1, 2, \ldots, N_{pin}$.

Elect N_{pin} nodes of the network to be pinned, one node each T seconds; been the node j that will be pinned at instants kT chosen by random. We can now define the function δ_i is $\delta_i = \delta_i(kT)$ defined as follows:

$$\delta_i = \begin{cases} 1, & \text{for } i = j, \\ 0, & \text{for } i \neq j, \ j = 1, ..., N_{pin}. \end{cases} \quad (5)$$

The controlled network (2) defined that way consists a decentralized fully adaptive node-to-node strategy that we propose in this work. We proof that the proposed strategy guarantees global asymptotic stability of the synchronized solution $\mathbf{x}_s$; and we show, via intensive simulation, that such strategy can present better performance than the usual node-to-node pinning strategy discussed in [16].

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