



Development of uninterrupted correlations of Prandtl number and its components to use in mathematical models of freezable heat pipes with expansion to super critical temperature region.

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***Abstract.** The development of continuous Prandtl number correlations and their constituent components is crucial to use in transient mathematical models of heat pipes (HPs), especially in satellite applications that require a very broad operational temperature range. Freezable HP is a new type of variable conductance pipe that operates when it really needs to cool electronic equipment at high temperatures and stop operation at low temperatures preventing sub-cooling. Any mathematical model shall be able to simulate transitions to and from the sub-cooled temperature region and even extend into the supercritical region. The accurate representation of such thermophysical properties as viscosity, thermal conductivity, and sensitive heat, which are components of the Prandtl number, over the entire temperature range without interruptions in its derivatives is pivotal in HP modeling. Seamless accommodates the transitions from solid to two-phase states and onward to the supercritical region is achieved by applications of Heaviside functions in the approximations.*

Keyword: Prandtl Number; Heat Pipe; Thermophysical Properties.

1. Introduction

The incorporation of heat pipes (HP) in satellite systems is necessary to achieve an efficient thermal management. In the harsh orbital environment, characterized by substantial temperature changes, the effective thermal management assumes a position of critical importance. HPs, rooted in the fundamental principles of phase-change heat transfer, assume a pivotal role in the spreading of surplus thermal energy, stemming from the operation of onboard electronic. HPs confer upon satellite systems an elevated level of endurance and operational efficiency. The first time presented the conception of gas-loaded variable-conductance heat pipes (VCHP) (MARCUS, 1972). As an example of promised VCHP application (EDELSTEIN; FLIEGER, 1978) showed the importance of using variable conductance HPs to extend the lifetime of battery allocated in a satellite; this battery is used to provide electrical energy to the satellite during the eclipse periods of the orbit.

The use of frozen water-filled HPs is a new conception of VCHP to use in satellites. In cold conditions the working liquid freezes and the process of thermal management stops, keeping



the heat of the equipment when its turn in safety mode. A point that deserves attention is to predict how this freeze/thaw cycles will impact the HP structure (ABABNEH; TARAU; ANDERSON, 2019) and how the melting-solidification transitions will occur.

Moreover, looking in the opposite temperature spectrum, cryogenics HPs are widely used to do thermal management of special instruments that need very low temperature range. For example, infrared sensors used in satellite optical systems require cooling to increase their measurement sensitivity (LEE et al., 2020), so a working fluid that achieve those requirements may be nitrogen, oxygen, or ethane, or another one, and its saturated temperature is so low that normal ambient temperature needed to HP start-up lies in the supercritical zone.

2. Methodology

Any mathematical model must effectively handle properties that critically rely on temperature. The operation of HPs in satellites typically is driven by a broad operational temperature range. Therefore, it's of utmost importance to possess accurate and precise property approximations across the entire temperature spectrum, encompassing solid, two-phase, and supercritical regions. When simulating transient HP (HP) modes, it is vital to employ continuous property relationships without disruptions in values and derivatives. Using tables and piecewise linear interpolations between tabulated values is deemed unacceptable. The most complex challenge arises when simulating specific transient HP modes, which in general requires a very short numerical time steps, in order of 0.0001 sec, and any irregularity or interruption in the property values or derivatives may provoke numerical instability in the mathematical model (VLASSOV; PANISSI; DE SOUSA, 2017).

Experimental studies and tests are an important part of the satellite's development, but they are usually expensive and time consuming. Because of that, numerical simulations play a vital role. Numerical models can be very reality-representative and simulate many phenomena and particular cases within a fraction of the experiment cost and time. A major part of property tables available in literature has limited range of temperature, but in case of HP used in satellites, a mathematical model must have adequate treatment of such temperature-dependent properties. Liquid Prandtl number and its components are used in momentum equations of liquid flow through HP capillary structure.

The Prandtl number is defined as the correlation between momentum diffusivity (ν) and the thermal diffusivity (α) as is possible to see in equation (1).

$$\text{Pr} = \frac{\nu}{\alpha} = \frac{\mu/\rho}{\kappa/(c_p\rho)} = \frac{c_p\mu}{\kappa} \quad (1)$$

The Prandtl number components are defined as following:

ν is momentum diffusivity or kinematic viscosity [m^2/s], α is thermal diffusivity [m^2/s], μ is dynamic viscosity [$\text{N s}/\text{m}^2$], κ is thermal conductivity [$\text{W}/\text{m}/\text{K}$], c_p is specific heat [$\text{J}/\text{kg}/\text{K}$]. As said above, these parameters are essential for HP numerical simulations, and they have unique behavior in temperatures close to triple point and close to critical point.



The approach that we are expecting to achieve in this work is to obtain a universal correlation for Prandtl number for water, that means that we expect use the same approximation with different coefficients also for other fluids. The important part of our approach is to present the final approximations in dimensionless variables, which do not depend on used unities in the properties.

Thermo-physical properties of water were studied widely and there are many published data and tables available for the using (for example, IAPWS, 1995).

To perform any thermal simulation, the Prandtl number is a key variable; it depends on temperature and its behavior is extremely not linear as the equations (2) and (3) can show.

About this property, (LILEY, 2005) suggested the following approximation:

$$\text{Pr}(T) = e^{-1.78023 + 501.834 / (T + 114.643)} \quad (2)$$

The applicability range starts in -20°C and ends in 50 °C.

FAGHRI in 2016, used polynomial approximation for components of Prandtl number.

$$\text{LN}[f(T)] = \alpha_0 + \alpha_1 T + \alpha_2 T^2 + \alpha_3 T^3 + \alpha_4 T^4 + \alpha_5 T^5 \quad (3)$$

Table 1 – Property coefficients by (FAGHRI, 2016)

	α_0	α_1	α_2	α_3	α_4	α_5
$c_{p,l}$ [kJ/kg/K]	1,435E+0	-3,223E-4	6,163E-6	-4,410E-8	2,097E-10	-3,040E-13
μ_l [Ns/m ²]	-6,353E+0	-3,154E-2	2,167E-4	-1,156E-6	3,747E-9	-5,219E-12
κ_l [W/m/K]	-5,822E-1	4,118E-3	-2,793E-5	6,562E-8	4,110E-11	-3,822E-13

The applicability range starts in 20°C and ends in 200 °C.

No approximations are available in literature for thermophysical properties analytical smoothed approximations without interruptions in a wide range of temperature, and not depended of unities.

For saturation zone, having this property initial table, it can be first transferred into not-dimensional form and then to search the best combination of different candidates of approximation functions of different types to build final algorithm in pseudo-code format from the obtained points. The temperature τ is also dimensionless, where 0 - means the triple point temperature, and 1 - critical temperature. As an option, the Prandtl number approximation can be obtained from the approximations of its components: viscosity, thermal conductivity, and sensible heat. This pseudo-code formal can be implemented in any programming language for using in transient mathematical models of HPs of different types.

3. Prandtl number components approximations

- **Liquid Specific Heat Capacity:**

For the saturation interval (i.e. two-phase zone, $0 < \tau < 1$), the original curve has a sharp climb as it approaches the critical temperature. It is difficult to obtain a unique function which approximates the entire zone within acceptable error. Therefore, we will resolve it with



application of an interruption point (τ_H) and join two different type approximation functions in the interval $\tau_H \leq \tau < 1$ by application of Heaviside function χ in equation (4). The best results gave the combination of following approximate functions:

$$\begin{cases} y_1 = a_B + b_B \tau + c_B \tau^2 + d_B \tau^3 + e_B \tau^4 + f_B \tau^5 + g_B \tau^6 \\ y_2 = \frac{h_B (\tau - \tau_{H1})^{i_B}}{1 - j_B (\tau - \tau_{H1})^{k_B}} \end{cases} \quad (4)$$

These functions are used in an additive way: when $0 < \tau < \tau_H$, then $y = y_1$; when $\tau_{H1} < \tau < \tau_{H2}$, then $y(\tau) = y_1(\tau) + y_2(\tau)$. This can be condensed in a unique correlation for our property (5).

$$\bar{c}_{p,L}(\tau) = y_1(\tau) + \chi(\tau - \tau_{H1}) y_2(\tau) \quad (5)$$

Table 2 - Obtained liquid specific heat coefficients for approximation in the saturation zone.

$a_B = -2E-6$	$d_B = -2.5295$	$g_B = 3.8797$	$j_B = 5E-1$
$b_B = -5.53E-2$	$e_B = 6.4967$	$h_B = 5E1$	$k_B = 1$
$c_B = 5.511E-1$	$f_B = -7.9777$	$i_B = 2$	$\tau_{H1} = 0.89$

• **Liquid Dynamic Viscosity:**

For this property we found the best result is achieved by the approach by substitution way to join different approximation functions at different parts of the saturation zone. To provide the smooth joining of two non-linear approximation functions, y_1 and y_2 , we inserted a 3rd order polynomial functions y_i , that is linear with respect the 4 coefficients (d_b , e_b , f_b and g_b); to create a final curve without ripples, expressed in equations (6) and (7).

$$\begin{cases} y_1 = a_B - b_B e^{-c_B \tau} \\ y_i = d_B + e_B \tau + f_B \tau^2 + g_B \tau^3 \\ y_2 = \frac{1}{2} (1 + \text{Tanh}^{j_B} (h_B (\tau - i_B))) \end{cases} \quad (6)$$

$$\mu_l(\tau) = \chi^{-1}(\tau - \tau_{H1}) y_1(\tau) + \chi(\tau - \tau_{H1}) \chi^{-1}(\tau - \tau_{H2}) y_i(\tau) + \chi(\tau - \tau_{H2}) y_2(\tau) \quad (7)$$

Table 3 - Liquid dynamic viscosity coefficient in saturation zone

$a_B = 9.79E-1$	$d_B = 2.42$	$g_B = -3.027E-2$	$j_B = 9.658E1$
$b_B = 9.49E-1$	$e_B = -4E-2$	$h_B = 1E14E1$	$\tau_{H1} = 8.4E-2$
$c_B = 1.072E1$	$f_B = 7.6E-1$	$i_B = -5.539E1$	$\tau_{H2} = 1.9E-1$

The smooth transitions were achieved by conditions of continuity of values and derivatives in the interruption points τ_{H1} and τ_{H2} .

• **Liquid Thermal Conductivity:**

Like the specific heat presented above, this property uses one Heaviside interruption by an additive way and can be expressed by the equations (8) and (9).



$$\begin{cases} y_1 = a_B + b_B \tau + c_B \tau^2 + d_B \tau^3 + e_B \tau^4 + f_B \tau^5 \\ y_2 = g_B (\tau - \tau_{H1})^{h_B} \end{cases} \quad (8)$$

$$\bar{k}_{L_B}(\tau) = y_1(\tau) + \chi(\tau_{H1}) y_2(\tau) \quad (9)$$

Table 4 - Liquid thermal conductivity coefficient in saturation zone

$a_B = 7E-3$	$c_B = 5.0444$	$e_B = 6.2103$	$g_B = 4E2$	$\tau_{H1} = 0.96$
$b_B = -1.92$	$d_B = -6.911$	$f_B = -2.075$	$h_B = 2$	

Summarizing, those three parameters above can be placed in graphic form in Figure 1.

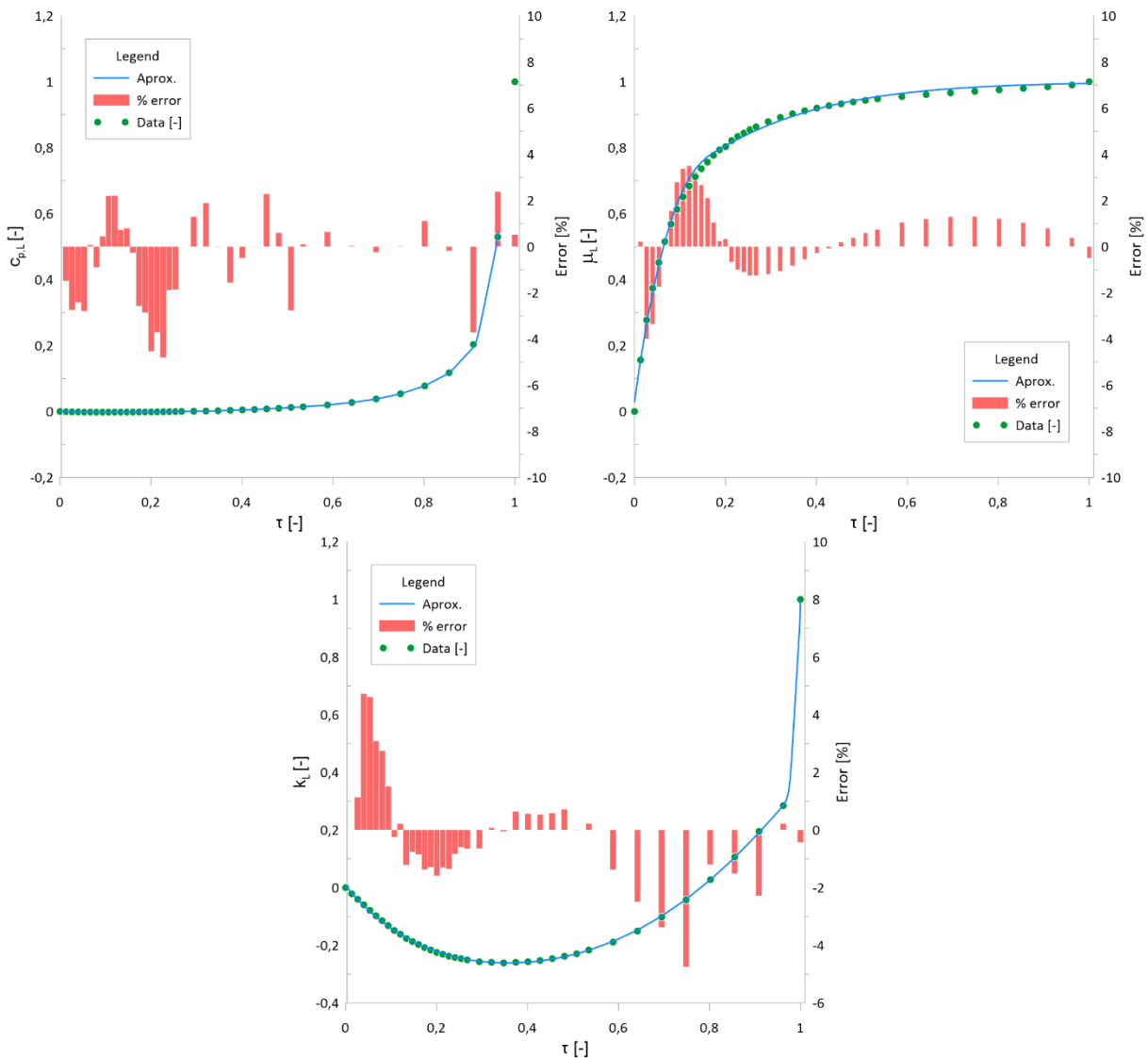


Figure 1 - Result of approximation parameters necessary to calculated Prandtl number components in saturation zone.

Following the general approach for all other properties, despite Pr number is already a dimensionless parameter, we introduce “reduced Pr number” (10) by the same way as for the Pr components:



$$\bar{\text{Pr}}_L(\tau) = \frac{\text{Pr}_L(T) - \text{Pr}_L(T_3)}{\text{Pr}_L(T_{cr}) - \text{Pr}_L(T_3)} \quad (10)$$

The input data for Pr number may be not available for some temperature intervals, for example below the triple point. To obtain the approximation to Prandtl number over entire temperature range, we use the relations between others dimensionless approximations by the following steps:

- Transforming the available dimensionless approximations of Pr components into dimensional values by inverse formulas;
- Building a table of values of Prandtl number, calculated from its components, as a function of temperature and pressure. Then add columns which calculate reduced Pr number as a function of dimensionless temperature and dimensionless pressure;
- Performing approximations of reduced Prandtl number to result in pseudo-code format.

It is important to expose that in the frozen zone, this Pr number is directly bond to the dynamic viscosity and the property behavior is mostly driven by the viscosity. Obviously, in the case of ice, this value is much higher when this is compared with the liquid value.

In this zone, the reduced Prandtl number can be approximated by a linear function (11):

$$\bar{\text{Pr}}_{L_A}(\tau) = a_A + b_A \tau \quad (11)$$

To provide a smooth transition between those two zones (sub-frozen and saturated) we use the interfacing equation (12) in a form a third-degree polynomial, similar as intermediate polynomial used for two-points Heaviside interruption by substitution way for the liquid dynamic viscosity:

$$\bar{\text{Pr}}_{L_{AB}}(\tau) = a_{AB} + b_{AB}\tau + c_{AB}\tau^2 + d_{AB}\tau^3 \quad (12)$$

The system of equation to calculate the coefficients in (12) will be of the format to interface tangency both curves and to make a continuous smooth link. We have four conditions to get four variables, which is solved analytically. Points of tangency were defined through optimization of the approximation result by minimization of maximal deviations from the property table points.

A similar approach was used for the interfacing of approximations around critical temperature T_{cr} ($\tau = 1$).

In super-critical zone, we correlate the pressure variation and temperature variation, resulting in an equation of two variables: dimensionless temperature and pressure (13).

The suggested format for super-critical zone is as follows:

$$\bar{\text{Pr}}_{L_c}(\tau, p) \Big|_{\tau > 1} = (a_B + b_B p) (c_B \tau^{d_B}) \quad (13)$$

The result is available in Figure 4.



3. Prandtl number resulting approximation

All analyzes are treated in the same way in terms of temperature. Initially, the analysis was done for $\tau < 0$ starting at $0 < \tau < 1$ and ending at $\tau > 1$.

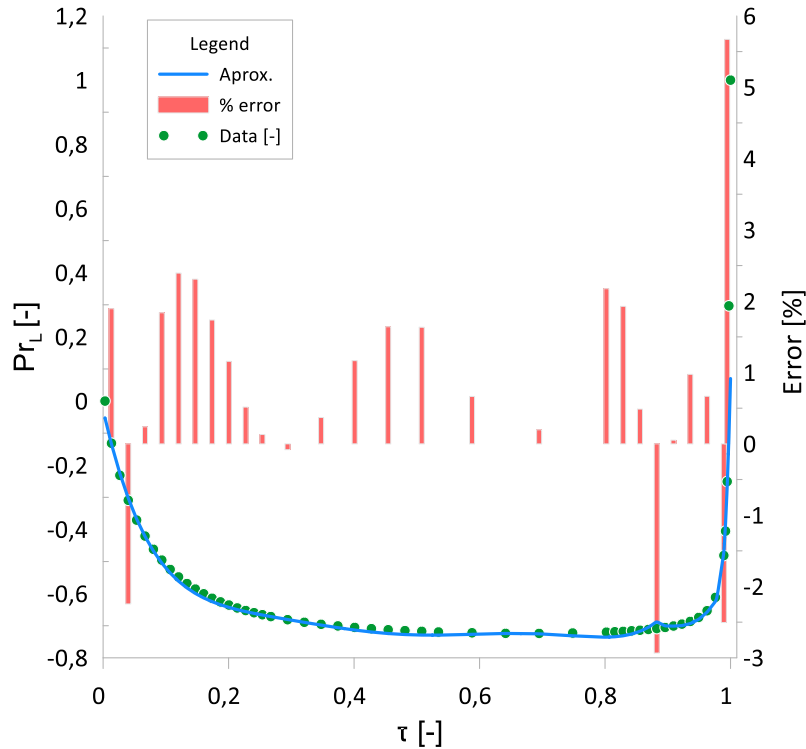


Figure 2 - Reduced Prandtl number final approximation in the two-phase zone

Therefore, we reached an approximation in a wider range of temperature, managing the approximation error in satisfactory levels (no greater than 5%).

Using a graphic representation, the interface around triple point $\tau=0$ is expressed in Figure 3.

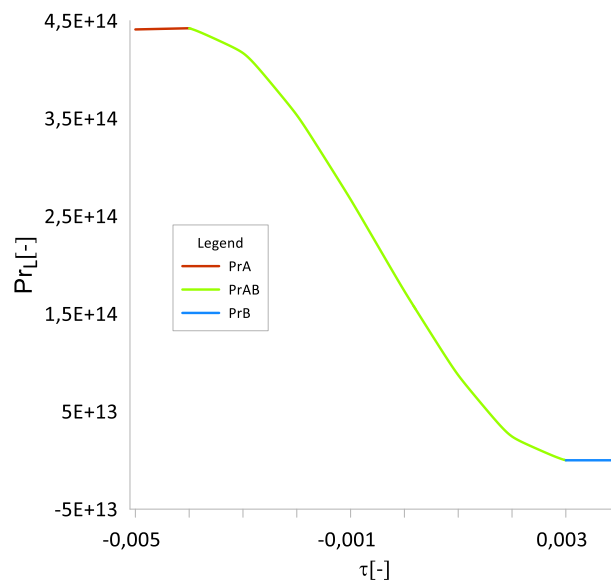


Figure 3 - Final result to the interface below sub frozen point.



Figure 4 shows results of obtained approximation in saturation and supercritical zones. Note, at the temperature above the critical, the Pr also becomes a function of pressure.

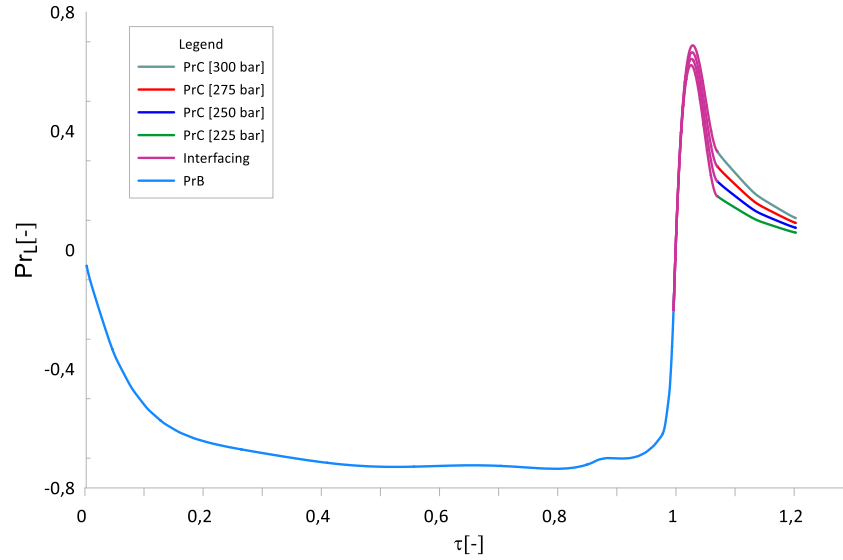


Figure 4 - Final result for Liquid reduced Prandtl Number approximation in entire temperature range.

The proposed algorithm for the approximation of the entire region includes also the zones below freezing point (Figure 3) and above critical point regions (Figure 2).

We present the results (14) of all approximation in the format of pseudocode analog in (CURRAN et al., 1995), which can be implemented by any programming language:

$$\bar{Pr}_L(\tau) = \begin{cases} \text{if } (\tau \leq \tau_1) & \text{then } \bar{Pr}_L(\tau) = a_A + b_A \tau \\ \text{if } (\tau_1 < \tau < \tau_2) & \text{then } \bar{Pr}_L(\tau) = a_{AB} + b_{AB} \tau + c_{AB} \tau^2 + d_{AB} \tau^3 \\ \text{if } (\tau_2 \leq \tau \leq \tau_3) & \text{then } \bar{Pr}_L(\tau) = \frac{c_P(\tau)\mu(\tau)}{\kappa(\tau)} \\ \text{if } (\tau_3 < \tau < \tau_4) & \text{then } \bar{Pr}_L(\tau) = a_{BC} + b_{BC} \tau + c_{BC} \tau^2 + d_{BC} \tau^3 \\ \text{if } (\tau \geq \tau_4) & \text{then } \bar{Pr}_L(\tau, p) = (a_B + b_B p)(c_B \tau^{d_B}) \end{cases} \quad (14)$$

where:

$$\tau = \frac{T - T_3}{T_{cr} - T_3} \quad \text{and} \quad p = \frac{P(T) - P(T_3)}{P(T_{cr}) - P(T_3)}$$

Table 5 - Resulting approximation parameters (Prandtl)

$a_A = 4.4707E14$	$b_{AB} = -9.245E16$	$a_{BC} = -7.398E3$	$d_{BC} = 6.317E3$	$c_C = 5.38E-1$	$\tau_2 = 3E-3$
$b_A = 1.2177E15$	$c_{AB} = 3.9173E18$	$b_{BC} = 2.106E4$	$a_C = -9.66E-1$	$d_C = -9.651$	$\tau_3 = 9.97E-1$
$a_{AB} = 1.7315E14$	$d_{AB} = 2.5536E21$	$c_{BC} = -1.998E4$	$b_C = 1.582$	$\tau_1 = 4E-3$	$\tau_4 = 1.06913$



To return to original Prandtl number from the obtained approximations, use inverse expression (15).

$$\Pr_L(T) = \Pr_L(T_3) + \text{aprx}(\bar{\Pr}_L(\tau))(\Pr_L(T_{cr}) - \Pr_L(T_3)) \quad (15)$$

where dimensionless temperature and pressure are defined above.

4. Conclusion

This work presents a methodology for approximations of water Prandtl number in the expanded temperature ranging from very low temperature (below triple point) to very high temperature (above critical point). Such property, as well as its 3 components, are the main parameters in any mathematical model of HPs and other two-phase heat transfer devices. Different types of approximations were combined in the equations using the interfacing uninterrupted method, having a continuous property results for the values as well as for the derivatives. The results are firstly presented in the format of pseudocode as a universal algorithm of a generalized dimensionless function. Such a function can be easily coded using any programming language and can be used in any mathematical model of HPs. For future work other fluids can be added in this algorithm to reach a whole group of the main working fluids which are widely used in aerospace engineering, such as ammonia, acetone, and others.

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Annex

Pr	Prandtl Number	T	Temperature [°C]
c	Heaviside Function	LN	Natural logarithm
n	Kinematic viscosity [m ² /s]	f(T)	Property function by temperature
a	Thermal diffusivity [m ² /s]	t	Dimensionless Temperature
m	Dynamic viscosity [N s/m ²]	P	Pressure [bar]
r	Density [kg/m ³]	y	Approximation function
k	Thermal Conductivity [W/m/K]	a, b, c, d	Variable
cp	Specific heat [J/kg/K]	e, f, g, h	Variable
		i, j, k	Variable

Subscribed

A	Frozen zone
B	Saturated zone
C	Super critical zone
H	Heaviside interruption
L	Liquid
CR	Critical
3	Triple Point
AB	Interfacing zone between frozen zone and saturated
BC	Interfacing zone between saturated zone and supercritical