NUMERICAL SOLUTION FOR MAGNETIC FLUID DROPLET

César F. C. Cristaldo[1]
cristaldo@lcp.inpe.br

Andris Bakuzis[2]
bakuzis@if.ufg.br

Fernando F. Fachini[1]
fachini@lcp.inpe.br

[1]Laboratório de Combustão e Propulsão - LCP
Instituto Nacional de Pesquisas Espaciais - INPE,
12630-000, Cachoeira Paulista, SP, Brazil

[2]Instituto de Física,
Universidade Federal de Goias,
74001-970, Goiânia, GO, Brazil

Abstract: *In this work we present an numerical solution for the heat transfer equation with sources, which describes the heating process of a droplet. Inside the liquid phase, there are magnetic nanoparticles (magnetic fluid). By applying an external alternating magnetic field, the nanoparticles rotate to align to the magnetic field and the rotation is against the viscous force. The result is heat generated by the viscous dissipation. The process enhances the droplet heating. The effect of the magnetic field is the uniform heating of the droplet. However, close to the droplet surface, under the influence of the heat conduction from the gas phase, there is a thin zone where the temperature profile depends on the spatial coordinate (thermal boundary layer). The energy conservation equation describing that thin zone is a heat equation with source term, in which we solve numerically by the finite difference method.*

Keywords: *Nanofluid, Droplet, Magnetic Power.*

1 INTRODUCTION

In spray problems, heat transfer between the planes determines the rate of the vaporization liquid. Thus, the heat transfer is the one of the processes that controls the fuel and oxidant mixture before burning (Meizhong, 2002). The control of droplet lifetime, heating time plus vaporization time, determines the combustion chamber length as well as the pollutant productions. The reduction of these times means smaller combustion devices burning eficiently fuels, i.e., maximum heat prodution and minimum pollutant production (Fachini, 2009).

Nanofluids have peculiar properties that are explored in many devices. Many studies have focused on thermal conductivity and on the viscosity of nanofluids. Nanofluids are compounds that have a base fluid and nanoparticles smaller than 100 nm dispersed in it. Nanofluids are expected to exhibit transport and thermodynamical superior properties when compared with conventional heat transfer fluids (Choi, 1995).

The nanofluids are a kind of functional fluid whose flow and energy transport can be controlled by adjusting an external magnetic field. These special fluids find a variety of applications in various fields such as packaging, electronic, mechanical engineering, aerospace, bioengineering, and thermal engineering (Xuan, 2007).

Since the diameter of the suspended magnetic particles is usually about 10 nm, these nanoparticles are considered to have a single magnetic moment (Xuan, 2007). Moreover, magnetic nanofluids expose to an alternating magnetic field can generate heat due to rotate of the magnetic dipoles fixed on the nanoparticles inside the fluid (Rosesweing, 2002). The Brownian motion always leads nanoparticles in random motion causing a misalignment of the magnetic dipoles.

The alternating magnetic field will align the magnetic dipole of the nanoparticles, the particle can rotate with the magnetic dipole (Brownian relaxation).When this occurs, the rotation of the nanoparticle causes friction with the surrounding fluid, generating heat.

2 MATHEMATICAL FORMULATION

The droplet is assumed to be spherical with radius $a(t)$ at time t, $a_0 = a(0)$ being the initial value, so that it has spherical symmetry in the liquid phase. It is assumed that the density ρ, the specific heat c_l and thermal conductivity of the liquid phase k_l are constant. The dimensionless variables used in this work are defined as:

$$t = t^*/t_c, \quad r = r^*/\bar{a}_0, \quad \theta = T/T_B \tag{1}$$

where $t_c = [a_0^2/(k_g/c_p\rho_\infty)](\rho_l/\rho_\infty)$ is the heating time of the droplet and T_B is the boiling temperature. The equation for energy dimensionless conservation (**?**) , is given by

$$\frac{\partial\theta}{\partial t} - \frac{A}{r^2}\frac{\partial\theta}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right) = P_m\frac{f^2\tau_m(\theta)}{1+(f\tau_m(\theta))^2} \tag{2}$$

where $A = c_pk_l/c_lk_g$, $f = 2\pi\bar{f}t_m^*$ is the dimensionless frequency, $\tau_m(\theta)$ the relaxation time a function of temperature and

$$P_m = \frac{\mu_0\chi_0H_0^2/2}{\rho_lc_lT_B}\frac{t_c}{t_m^*} \tag{3}$$

where μ_0 is the magnetic permeability, χ_0 is the magnetic susceptibility, H_0 is the magnetic field amplitude and t_m^* is the relaxation time (Maenosono and Saita, 2006). P_m represents the ratio of the power magnetic to the thermal power . The model proposed in this paper considers the case $P_m \gg 1$. This means that the power magnetic is much larger than the thermal power due to the heat conduction from the gas phase.

To solve the problem Eq.(2), the time and the length must be rescaled.

For $P_m \gg 1$ the term predominant in Eq.(2) is the source term, indicating that in the most part of the droplet the temperature increases uniformly, except in a thin zone near the surface of the drop where the temperature increases rapidly (fig. 1-a). To observe the temperature evolution is necessary to rescale the time according $\tau = tP_m$ since the appropriate time scale is $t \sim P_m^{-1}$. Beside that, it is also necessary to scale the spatial coordinate around the droplet surface to follow the variation of the temperature that occurs in a thin zone. The rescale is $r = a - \varepsilon x$. The length of order ε specifies the thermal boundary layer problem, Fig. 1-b.

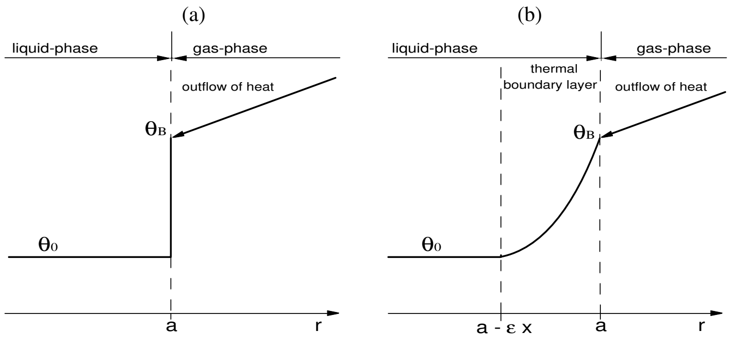

Figure 1: Temperature profile: (a) length scale $O(1)$. (b) lenght scale $O(\varepsilon)$.

The Eq.(1) in the appropriate scale is written

$$\frac{\partial\theta}{\partial\tau} - \frac{A}{P_m\varepsilon^2}\frac{\partial^2\theta}{\partial x^2} = \frac{f^2\tau_m(\theta)}{1+(f\tau_m(\theta))^2}, \tag{4}$$

where $\varepsilon = \sqrt{A/P_m}$ is chosen to describe the transient non uniform variation of the temperature. Note that, in these new variables the source term of Eq.(4) is the $O(1)$. An analysis on Eq.(4), for $f \gg 1$ the time scales would $\tau \sim 1$ and for $f \ll 1$, $\tau \sim f^{-2}$.

Due to Brownian processes, the effective relaxation time $\tau_B = 3\eta V_H/(kT)$ (Rosesweing, 2002), where η the viscosity, K the Boltzman constant, T the temperature and V_H the hydrodynamic volume of the particle. In this paper, it's considered that $\tau_m = \tau_B$, i.e., the energy dissipation is due only for Brownian mechanism. The relaxation is written as $\tau(\theta) = 1/\theta$ (Fachini, 2009). So the Eq.(4) is written as

$$\frac{\partial\theta}{\partial\tau} - \frac{\partial^2\theta}{\partial x^2} = \frac{f^2\theta}{\theta^2+f^2} \tag{5}$$

The boundary conditions, coupling gas and liquid phases, are:

$$\left.\frac{\partial \theta}{\partial x}\right|_{x=+\infty} = 0 \tag{6}$$

$$k_g \left.\frac{\partial \theta}{\partial x}\right|_{x=0^+} - k_l \left.\frac{\partial \theta}{\partial x}\right|_{x=0^-} = \lambda l \tag{7}$$

The initial condition:

$$\theta = \theta_0, \quad x < a, \quad \tau = 0 \tag{8}$$

were the nondimensional vaporization rate is $\lambda = \dot{m}c_p/(4\pi\bar{a}_0)$ and $l = L/c_p T_\infty$ is the latent heat. It's considered the hypothesis of all heat transported from the gas phase is used only of heat up the droplet, ie, the vaporization is negligible ($\lambda \ll 1$). The next step is to solve the problem.

3 SOLUTION

The problem is a nonlinear heat equation with Neumann condition on the boundary and initial condition. We impose the condition that the process of heat generated by the nanoparticles produces an uniform temperature profile within the droplet (except in a thin zone near the surface) and outside the droplet consider a known heat flux. As seen in Fig.1.

Equation (5) is solved numerically by the finite difference method, with a forward difference for the time derivative and a second-order central difference for the space derivative at position x_j. The numerical result reproduces the evolution of the dimensionless temperature for the several values of frequencies.

The problem of droplet heating has an restriction, temperature value is below the boiling temperature, $\theta_0 \leq \theta < 1$. According to Eq.(5), the boiling temperature is reached when $\tau \sim 1$ (time of heating of the droplet) in the droplet surface. Remembering that $t^*/t_c \sim 1/P_m$ for $f \gg 1$ and $t^*/t_c \sim 1/(f^2 P_m)$, therefore, the time required to monitor the temperature in the thermal boundary layer for $f \gg 1$ is less than for $f \ll 1$.

This paper estimates the thickness of the thermal boundary layer δ. By remembering that the change of coordinates leads to $r^* = a - \varepsilon\delta$, that is of order unity for $\delta = O(1)$. However, for $P_m \sim 1$, $\varepsilon \sim 1$ and the thermal boundary layer extends to the whole droplet. The length the thermal boundary layer was estimated to increase by 5% from the temperature inside the droplet.

4 RESULTS

The solution of Eq.(5) is presented below. We use the following notations $f < 1$ (low frequency) and $f > 1$ (high frequency). Figure 2 represents the evolution temperature up to $\theta = 1$ at $x = 0$ for different frequency f. Note that the increasing frequency, the droplet surface readily reaches the temperature of boiling. This can clearly be seen in Fig.3, and note also that for high-frequency heating time is independent of frequency.

Figure 4 represents the thickness of the thermal boundary layer as a function of frequency. Note that the thickness of the thermal boundary layer decreases with the increases of the frequency to a certain limit, this can be confirmed by the Fig. 2 showing that for high frequency the profile of the temperature does not change.

Figure 5 shows the temperature profile as a function of frequency for different values of time at the surface of the droplet. We observed that for $f = 100$ the droplet reaches the maximum temperature, while that for low frequency would require longer time to reach the boiling temperature. Note that below and above a certain frequencies, the temperature is practically independent of frequency, as confirmed by Eq.(5) with the time scale appropriate.

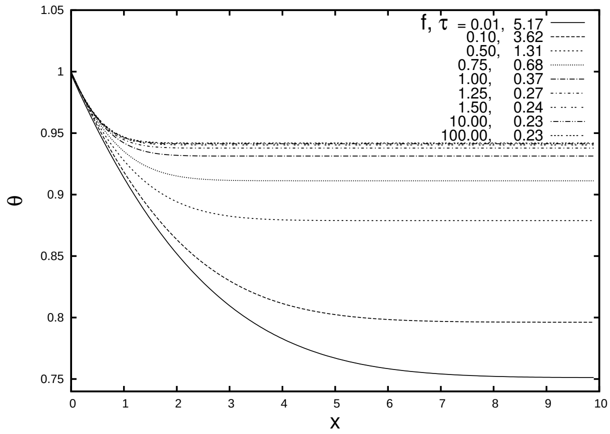

Figure 2: Evolution of temperature for different value of the frequency in $\tau = \tau_B$.

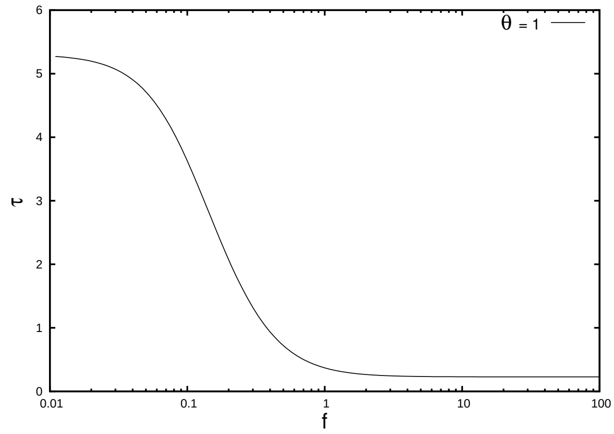

Figure 3: Heating time of droplet surface as a function of the frequency.

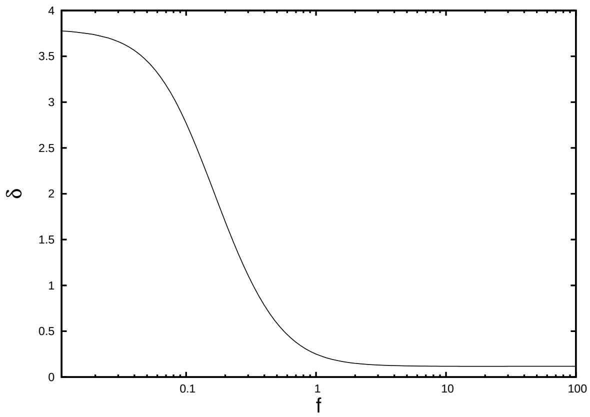

Figure 4: Thermal boundary layer thickness as a function of the frequency.

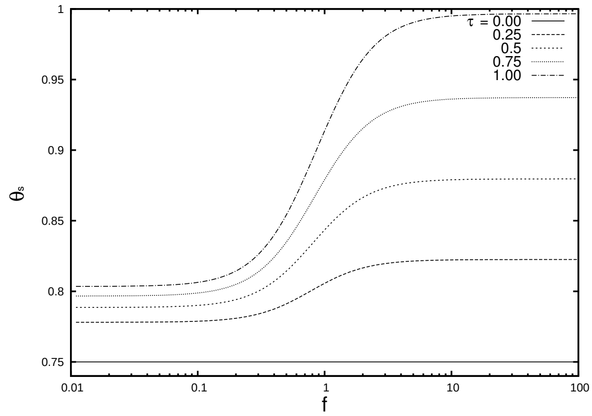

Figure 5: Evolution of the droplet surface temperature as a function of the frequency .

5 CONCLUSION

The paper shows the influence of magnetic nanoparticles dispersed in liquid on the heating process of droplets. Also, the developed model can help for the planning of experimental studies. Future studies can take into account other heat source mechanism, such as the Nel mechanism.

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