# Cosmology from Kaluza-Klein gravitational model

P. H. R. S. Moraes and O. D. Miranda

Citation: AIP Conf. Proc. **1483**, 435 (2012); doi: 10.1063/1.4756990
View online: http://dx.doi.org/10.1063/1.4756990
View Table of Contents: http://proceedings.aip.org/dbt/dbt.jsp?KEY=APCPCS&Volume=1483&Issue=1
Published by the AIP Publishing LLC.

# Cosmology from Kaluza-Klein Gravitational Model

P.H.R.S. Moraes and O. D. Miranda

*INPE - Instituto Nacional de Pesquisas Espaciais - Divisão de Astrofísica*
*Av. dos Astronautas 1758, São José dos Campos, 12227-010 SP, Brazil*

**Abstract.** In this work we present a cosmological model derived from Kaluza-Klein theory, identifying the functional form of the parameter associated to the fifth dimension in such a way that it produces a solution that mimetizes the effects of the cosmological constant in the scope of General Relativity.

**Keywords:** cosmology, dark energy, extra dimensions
**PACS:** 04.50.Cd; 04.50.Kd; 95.36.+x; 98.80.-k

## INTRODUCTION

We know that the universe is expanding in a accelerated way. The cause of this acceleration, named dark energy, is one of the greatest mysteries in cosmology. In General Relativity (GR) framework , the existence of a cosmological constant, which is related to the quantum vacuum energy, in Einstein Equations would bring this property to the current universe, but questions like: what does the cosmological constant represents?; what is the fundamental reason of its inception in Einstein Equations?; would it be the correct model to explain the cosmic acceleration of the universe?, can not be neglected.

The cosmology provide us for the density parameter of the cosmological constant $\Lambda$, $\Omega_\Lambda \sim 0,73$ or $\rho_\Lambda \sim 0,73\rho_c$ [1], where $\rho_c$ is the critical density of the universe ($\rho_c = 3H_0^2/8\pi G$). In this way, in physical units, we have $\rho_\Lambda \sim 10^{-47}GeV^4$. However, from theoretical physics, the value obtained for the vacuum energy [2] is $\rho_\Lambda \sim 10^{71}GeV^4$.

Thus, although the $\Lambda$CDM model ($\Lambda$ Cold Dark Matter) is successful in explaining the supernovae Ia luminosity distances [3] [4], X-ray spectrum of clusters of galaxies [5] and Baryon Acoustic Oscillations (BAO) [6] [7], the discrepancy above between the values of the vacuum density energy leads us to examine others models of universe.

One alternative is to consider extra dimensional models. Extra dimensional models have been used as an alternative to the inconsistencies of GR, and, in this way, can constitute an important outcome to the dark energy problem.

Others models of dark energy which will not be appreciated in this work are: $\phi$CDM model [8], Chaplygin gas [9], CCDM model [10], XCDM model [11], $\Lambda(t)$CDM model [12], inhomogeneous models [13], $f(R)$ theories [14] and kinematic models

*The Sixth International School on Field Theory and Gravitation-2012*
AIP Conf. Proc. 1483, 435-440 (2012); doi: 10.1063/1.4756990

[15].

In this work we present a cosmological model derived from Kaluza-Klein theory, identifying the functional form of the parameter associated to the fifth dimension in such a way that it produces a solution that mimetizes the effects of the cosmological constant in the scope of General Relativity.

## KALUZA-KLEIN GRAVITATIONAL MODEL

Created by Theodor Kaluza and first published in 1921, Kaluza-Klein model shows that GR in five dimensions contains both Einstein's four-dimensional gravitational theory and Maxwell's theory of electromagnetism. To do so, Kaluza imposed an artificial restriction on the coordinates (so-called cylindrical condition), which consists on the annulment of all derivatives with respect to the fifth dimension. Oscar Klein's contribution was to make this restriction less artificial in 1926, suggesting a compatification of the fifth dimension.

### Kaluza-Klein mechanism

Kaluza has demonstrated that GR when interpreted as a vacuum 5D theory contains four-dimensional GR in the presence of electromagnetic field [16], together with Maxwell's electromagnetism. To do so, Kaluza supposed that:

- his model should maintain Einstein's vision that nature is purely geometric;
- GR mathematics is not modified, just extended to five dimensions;
- there is no physical dependence on the fifth dimension (cylindrical condition).

The first presupposition implies that Einstein equations must be written as

$$G_{AB} = 0, \tag{1}$$

where $A$ and $B$ run from 0 to 4, or equivalently

$$R_{AB} = 0, \tag{2}$$

since

$$G_{AB} = R_{AB} - \frac{1}{2} g_{AB} R, \tag{3}$$

in this way, everything depends (only) on the properties of the metric.

Considering a four dimensional metric $g_{\alpha\beta}$ , Kaluza extended this metric to force fifth dimension to induce eletromagnetism. To do so, the $g_{\alpha 4}$ components of the metric are connected with the electromagnetic potential $A_{\alpha\beta}$ , and $g_{44}$ is defined in terms of the scalar field $\phi$. So, the 5D metric is

$$g_{AB} = \begin{pmatrix} g_{\alpha\beta} + \kappa^2\phi^2 A_\alpha A_\beta & \kappa\phi^2 A_\alpha \\ \kappa\phi^2 A_\beta & \phi^2 \end{pmatrix}, \tag{4}$$

where $\kappa$ is a constant inserted in order to obtain the correct dimension of the metric (we choose $c = \kappa = 1$).

If we apply cylindrical condition, from metric (4) we get the following field equations in four dimensions:

$$G_{\alpha\beta} = \frac{\kappa^2\phi^2}{2} T^{EM}_{\alpha\beta} - \frac{1}{\phi}[\nabla_\alpha(\partial_\beta\phi) - g_{\alpha\beta}\Box\phi], \tag{5}$$

$$\nabla^\alpha F_{\alpha\beta} = -3\frac{\partial^\alpha\phi}{\phi} F_{\alpha\beta} \tag{6}$$

and

$$\Box\phi = \frac{k^2\phi^3}{4} F_{\alpha\beta}F^{\alpha\beta}, \tag{7}$$

where

$$T^{EM}_{\alpha\beta} = \frac{g_{\alpha\beta}F_{\gamma\delta}F^{\gamma\delta}}{4} - F^\gamma_\alpha F_{\beta\gamma} \tag{8}$$

is the electromagnetic energy-momentum tensor and

$$F_{\alpha\beta} \equiv \partial_\alpha A_\beta - \partial_\beta A_\alpha. \tag{9}$$

We also assume

$$\kappa\phi = 4\sqrt{\pi G}. \tag{10}$$

In this way, Equations (5) and (6) become

$$G_{\alpha\beta} = 8\pi G\phi^2 T^{EM}_{\alpha\beta} \tag{11}$$

and

$$\nabla^\alpha F_{\alpha\beta} = 0. \tag{12}$$

Equation (11) is Einstein equation and equation (12) is one of the GR Maxwell's equations $(\nabla_\mu F_{\mu\nu} = kJ_\nu)$ in the absence of eletric current. Equations (11) and (12) yields the conclusions:

- Maxwell's equations are contained in 5D Einstein equations for vacuum;
- 5D Einstein equations for vacuum induce to Einstein equations with matter in 4D.

## ANALYSIS OF THE COSMOLOGY FROM KALUZA-KLEIN MODEL

Starting from the specific case of Kaluza-Klein model where the fifth dimension is not compactified and so the derivatives with respect to its coordinate do not vanish, the idea of this analysis is to insert a fifth dimension term with structure $\alpha^2 dl^2$, in an line element like the Friedmann-Robertson- Walker (FRW), where $\alpha$ is a time-dependent parameter whose dimension encloses the correct line element dimension and $d$l has a space-like signature. Therefore, our line element is

$$ds^2 = dt^2 - a(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sen^2\theta d\phi^2\right) - \alpha(t)^2 dl^2, \tag{13}$$

where $c = 1$, $a(t)$ is the scale factor of the universe and we consider the 5D space-time to be flat ($k = 0$).

From FRW cosmology, the equation that gives the universe density evolution, with a cosmological constant is

$$8\pi G\rho(t) = 3\left(\frac{\dot{a}}{a}\right)^2 - \Lambda. \tag{14}$$

Following the procedure of [17], we obtain an analogue of Equation (14) for the 5D case, in order to verify if the treatment of the universe in five dimensions could, somehow, mimetize the cosmological constant effects of this equation.

We know that, in four dimensions

$$G_{ab} = kT_{ab}. \tag{15}$$

Using (1), we can rewrite (15) as:

$$G_{ab} - G_{AB} = kT_{ab}. \tag{16}$$

Therefore, all we need to find the matter properties is the Einstein tensor in 4D and 5D (remind that the (00) component of the energy-momentum tensor is $\rho$). The (00) components of these tensors are, respectively:

$$G_0^0 = 3\left(\frac{\dot{a}}{a}\right)^2, \qquad \text{and} \qquad G_0^0 = 3\left(\frac{\dot{a}}{a}\right)^2 + 3\frac{\dot{a}}{a}\frac{\dot{\alpha}}{\alpha} \tag{17}$$

Applying (17) in the (00) components of (16), we have:

$$8\pi G\rho(t) = -3\left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{\alpha}}{\alpha}\right). \tag{18}$$

The functional form of the parameter $\alpha(t)$ comes from a toy-model obtained equaling (14) with (18), which means say that $\alpha(t)$ must mimetize the cosmological constant effects. Therefore we can write

$$-3\left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{\alpha}}{\alpha}\right)=3\left(\frac{\dot{a}}{a}\right)^2-\Lambda. \tag{19}$$

The solution of (19) for $\alpha$ is:

$$\frac{\alpha}{\alpha_0}=(1+z)exp\left\{-\frac{\Lambda}{3H_0^2}\left[\frac{4}{3}ln(1+z)-\frac{4}{9}ln(z^3+3z^2+3z+1)\right]\right\}. \tag{20}$$

Therefore in this toy-model the parameter $\alpha$, associated to the fifth dimension can be characterized by the present Hubble parameter ($H_0$) and cosmological constant values.

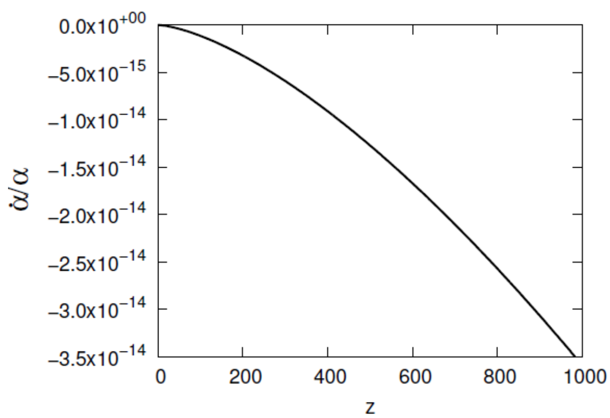


**FIGURE 1.** Evolution of the function $\dot{\alpha}/\alpha_0$, associated with the fifth dimension on Kaluza-Klein model, versus redshift.

Figure 1 presents the evolution of $\dot{\alpha}/\alpha$ in Kaluza-Klein model. The substitution of this result in (18) produces exactly the same evolution produced by ΛCDM model.

## FINAL REMARKS

We started a study of the Kaluza-Klein gravitational model. First, we derived from it a toy-model and we have shown that the treatment of the universe as a five dimensional space-time can mimetize the effects of dark energy. A physical interpretation of the fifth dimension and its particular form as taken in this toy-model are important questions that will be treated in the extension of this work.

## REFERENCES


1. J. Weiland, et. al., *Astrophys.J.Suppl.*, **192**, 19 (2011).

2. S. Weinberg, *Reviews of Modern Physics*, **61**, 1 (1989).
3. A. G. Riess, et. al., *The Astronomical Journal*, **116**, 1009 (1998).
4. G. Perlmutter, et. al., *The Astrophysical Journal*, **517**, 565 (1999).
5. S. W. Allen, R. Schmidt, K. Ebeling, A. C. Fabian and L. Van Speybroeck, *Monthly Notices of the Royal Astronomical Society*, **353**, 457 (2004).
6. D. Eisenstein, I. Zehavi and D. W. Hogg, *The astrophysical journal*, **633**, 560 (2005).
7. W. J. Percival, et. al., *Monthly Notices of the Royal Astronomical Society*, **401**, 2148 (2010).
8. P. J. E. Peebles and B. Ratra, *Astrophysical Journal*, **325**, L17 (1988).
9. N. Bilic, G. B. Tupper and R. D. Viollier, *Physics Letters B*, **535**, 17 (2002).
10. G. Steigman, et. al., *Journal of Cosmology and Astroparticle Physics*, **6**, 33 (2009).
11. M. S. Turner and M. White, *Physical Review D*, **56**, r4439 (1997).
12. M. Ozer and M. O. Taha, *Physical Letters B*, **171**, 363 (1986).
13. M. Célérier, *New Advances in Physics*, **1**, 29 (2007).
14. D. N. Vollick, *Classical and Quantum Gravity*, **21**, 3813 (2004).
15. J. V. Cunha, *Physical Review D*, **79**, 4 (2009).
16. J. Overduim and P. Wesson, *Physical Reports*, **283**, 303 (1997).
17. P. Wesson, *World Scientific Publishing*, 1999.