Control of Spacecrafts Using Gravitational Forces

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Abstract: - The objective of the present paper is to perform a computational study of the ballistic gravitational capture in a dynamical model that has the presence of three bodies. In particular, the Neptune-triton-Spacecraft system is considered. This phenomenon is explained in terms of the integration of the perturbing forces with respect to time. Numerical simulations are performed to show the trajectories of the spacecraft. There are also graphs that has the purpose to obtain the minimum consumption of fuel for space maneuvers.

Key-Words: - Space Trajectories, Applied Mathematics, Gravitational Capture, Astrodynamics, Space Travel, Applied Computational Analysis.

1 Introduction

Neptune is the eighth planet of the solar system and it is the fourth largest planet in size and it is the third largest in terms of mass. It has thirteen moons, and the largest is Triton. A temporary gravitational capture happens when a spacecraft (or any particle with despicable mass) changes from a hyperbolic orbit around of a heavenly body to an elliptic orbit without the use of any propulsive force acting in the system. The force responsible for this change in the orbit of the spacecraft is the gravitational force of another celestial body. In this work it was made a numeric study of the gravitational capture.

The numeric stydy of gravitational capture using the restricted problem of three bodies in the system Earth and Moon is made in the references [1] until [5].

The mathematical model used to analyze the phenomenon of gravitational capture is the restricted circular problem of three bodies. In the references [6] and[7] we have been describing this mathematical model.

We used as the two primary bodies Neptune and Triton. In this text we used the criterion of gravitational capture used by [7] and [3]. Then, we calculated, using the energy of two bodies, to verify if an orbit is or is not of the gravitational capture type.

In the reference [8] it is made the study of gravitational capture the restricted mathematical model of the three bodies elliptic.

In the reference [9] we have one analyses of gravitational capture using the problem of four bodies, it is also calculated the varieties capture in [9].

2 Mathematical Models

The problem of three bodies with the two hypotheses shown below is called restricted circular problem of three bodies.

First hypothesis. It is considered the existence of two bodies with significant masses moving in circular orbits around the mutual center of mass.

Those two bodies are called primaries.

Second hypothesis. The third body has negligible mass compared with the primaries. In Figure 1 it is shown the motion of the two primaries.

The goal is to study the motion of a third body, with negligible mass, under the gravitational attractions of the two bodies with significant mass.

The planar equations of motion of the space vehicle can be written in terms of the canonical system of units, that are the units that we obtain by dividing all the distances by the distance between the two primaries and dividing all the masses by the total mass of the two primaries. It is also defined that the angular velocity of the primaries is unitary.

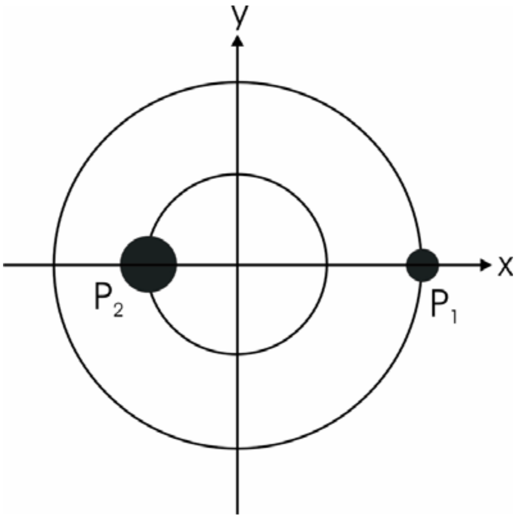

Figure 1. Move of the two primaries.

In the present paper, the primaries will be Neptune and Triton. The masses and distances of the Neptune and Triton are: mass of Neptune, $M_N = 1.02X10^{26}$ kg, mass of Triton, $M_T = 2.14X10^{22}$ kg. Neptune-Triton distance is d = 354670 km.

Then, the masses of Neptune and Triton, in the canonical system of units are:

Mass of Neptune $\mu_N = \frac{M_N}{M_N+M_T} = 0.99979024010$.

Mass of Triton $\mu_T = \frac{M_T}{M_N+M_T} = 0.0002097599131$.

The circumferences described by Triton and Neptune has radius μ_N and μ_T, respectively. (ξ,η), (ξ_N,η_N) and (ξ_T,η_T) are the sidereal coordinates of the space vehicle, Neptune and Triton

We have the equations of motion for Neptune in the inertial system as:

$$\begin{aligned} \xi_N &= -\mu_T \cos(t) \\ \eta_N &= -\mu_T \sin(t) \end{aligned} \quad (1)$$

The equations of motion for Triton in the inertial system is:

$$\begin{aligned} \xi_T &= \mu_N \cos(t) \\ \eta_T &= \mu_N \sin(t) \end{aligned} \quad (2)$$

The distance of the space vehicle to Neptune is:

$$r_1 = \sqrt{(\xi-\xi_N)^2 + (\eta-\eta_N)^2} \quad (3)$$

The distance of the space vehicle to Triton is:

$$r_2 = \sqrt{(\xi-\xi_T)^2 + (\eta-\eta_T)^2} \quad (4)$$

Therefore, we have the equations of motion of the space vehicle in the inertial system as:

$$\begin{aligned} \frac{d^2\xi}{dt^2} &= -\mu_T \frac{(\xi-\xi_T)}{r_1^3} - \mu_N \frac{(\xi-\xi_N)}{r_2^3} = -\frac{\partial V}{\partial \xi} \\ \frac{d^2\eta}{dt^2} &= -\mu_T \frac{(\eta-\eta_T)}{r_1^3} - \mu_N \frac{(\eta-\eta_N)}{r_2^3} = -\frac{\partial V}{\partial \eta} \end{aligned} \quad (5)$$

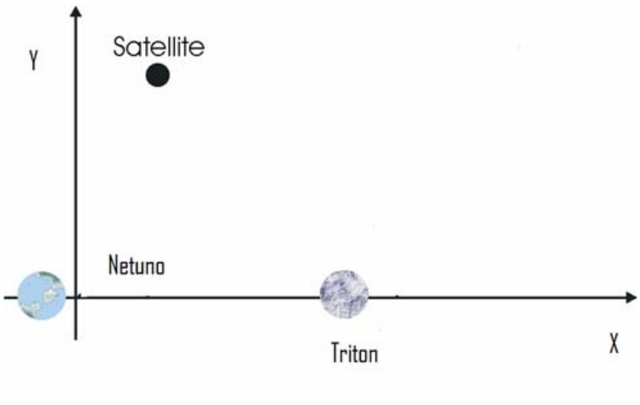

Figure 2 - Initial position of the three bodies.

We now need to introduce a system of coordinates that rotates together with the two primaries (Fig. 3) with origin in the center of mass of the primaries and with the same angular velocity of them. Be (x,y) the coordinates of the space vehicle in this system, also called synodic. The equations that convert the coordinates of the fixed system for the rotating system are:

$$\begin{pmatrix} \xi \\ \eta \end{pmatrix} = \begin{pmatrix} \cos(t) & -\sin(t) \\ \sin(t) & \cos(t) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad (6)$$

The coordinates of Triton and of Neptune in this new system are given by: $x_T = \mu_N$, $y_T = 0$, $x_N = -\mu_T$ e $y_N = 0$.

The distance r_1 of the space vehicle to Neptune in this new system is:

$$r_1^2 = (x - x_N)^2 + y^2 \quad (7)$$

The distance r_2 of the space vehicle to Triton in this new system is:

$$r_2^2 = (x - x_T)^2 + y^2 \quad (8)$$

The equations of motion of the space vehicle in this synodic system are:

$$\begin{cases} \frac{d^2x}{dt^2} - 2\frac{dy}{dt} = x - \mu_T \frac{x - x_T}{r_1^3} - \mu_N \frac{x - x_N}{r_2^3} = -\frac{\partial V}{\partial x} \\ \frac{d^2y}{dt^2} + 2\frac{dx}{dt} = y - \mu_T \frac{y}{r_1^3} - \mu_N \frac{y}{r_2^3} = -\frac{\partial V}{\partial y} \end{cases} \quad (9)$$

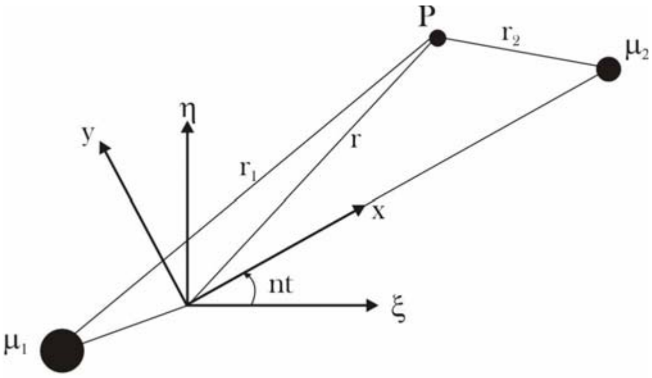

Figure 3 Synodic System.

3 Gravitational Capture

Figure 4 shows a trajectory that ends in gravitational capture. To define gravitational capture it is necessary to use some basic concepts of the problem of two bodies. We will call C_3 the double of the sum of the kinetic and potential energy of the problem of two bodies, the space vehicle and Triton, that is given by: $C_3 = V^2 - \frac{2\mu_T}{r}$ where r and V are, respectively, the distance and the velocity of the space vehicle with respect to the Triton, and μ_T is the gravitational parameter of the Triton.
If we consider only two bodies (Triton and the space vehicle), C_3 is constant, if only gravitational forces are considered. We will describe the orbits of the space vehicle for values of C_3 according to the classification: (i) If $C_3 > 0$, we have hyperbolic orbits, (ii) If $C_3 = 0$, we have parabolic orbits, (iii) If $C_3 < 0$, we have elliptic orbits.
We defined C_3 as being the double of the energy of the system Triton-vehicle. Unlike what happens in the problem of two bodies, C_3 is not constant in the bi-circular problem.

Then, for some initial conditions, the space vehicle can alter the sign of the energy from positive to negative or from negative to positive. When the variation is from positive to negative it is called a gravitational capture orbit. The opposite situation, when the energy changes from negative to positive, is called gravitational escape.
We describe the numeric methodology below.

1) A Runge-Kutta 7-8 integrator was used, programmed in the FORTRAN language.

2) We integrated the equations of motion of the space vehicle in the sidereal system.

3) The initial conditions are obtained in the following way. We consider the Triton in the origin of the system and Neptune with coordinates (-1, 0). The starting point of each trajectory is at a distance of 100 km from the surface of the Triton. To specify the initial position completely it is necessary to know the value of one more variable. The variable used is the angle α, that is the position of Triton. This angle is measured starting from the Neptune-Triton line, in the counterclockwise sense, starting from the opposite side of Neptune. The magnitude of the initial velocity is calculated from the initial value of energy. The direction of the velocity vector of the vehicle is chosen as being perpendicular to the line that links the space vehicle to the center of the Triton, appearing in the counterclockwise direction for the direct orbits and in the clockwise direction for the retrograde orbits.
The orbit is considered of capture when the particle reaches the distance of 0.26 canonical units from the center of Triton in a time smaller than 5 canonical units. Figure 4 shows the point P, where the space vehicle escapes from the sphere of influence. The angle that defines this point is called the angle of the entrance position and the Greek letter β is used to define it.During the numeric integration the step of time is negative, therefore the initial conditions are really the final conditions of the orbit after the capture.

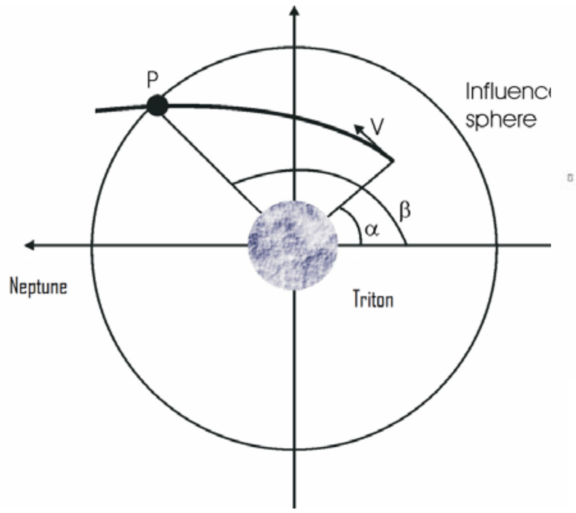

Figure 4 Initial conditions of the space vehicle

4 Numerical Results

All the graphs below are made in the synodic system. We will calculate orbits of direct and retrograde capture. In Figure 5 we have orbits of retrograde captures with the initial conditions $C_3 = -0.001$, $\alpha = 30^0, 45^0$ and 90^0. The orbit in red is for $\alpha = 30^0$, blue for α = 90^0 and green for α = 45^0. In Figure 6 we have the variation of the energy as a function of time, to observe that a change of the sign of the energy exists.

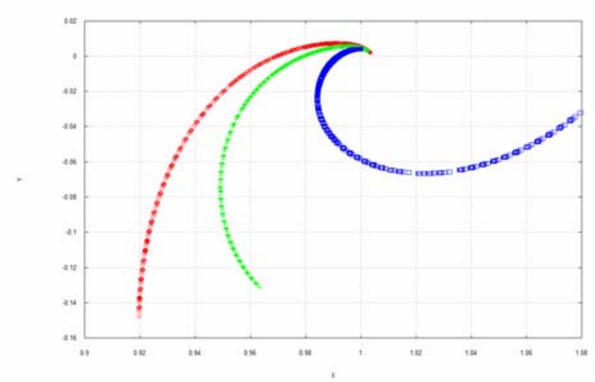

Figure 5 Retrograde orbits $C_3 = -0.001$, $\alpha = 30^0, 45^0$ and 90^0.

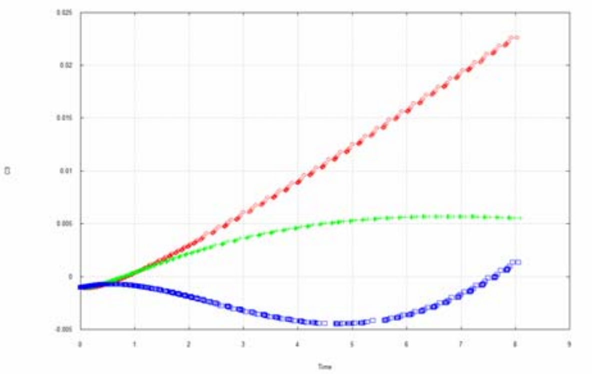

Figure 6 Energy as a function of the time

We defined as the time of gravitational capture the instant where there is the change of the sign of the energy. The time of gravitational capture is approximately 7 and a half days for the orbit $\alpha = 90^0$ and approximately one day for the other ones.

In Figure 7 we have orbits of direct captures with the initial conditions $C_3 = -0.001$, $\alpha = 30^0, 45^0$ and 90^0. The orbit in red is for $\alpha = 30^0$, blue for α = 90^0 and green for α = 45^0.

In Figure 8 we have the variation of energy as a function of time, to observe that a change in the sign of the energy exists.

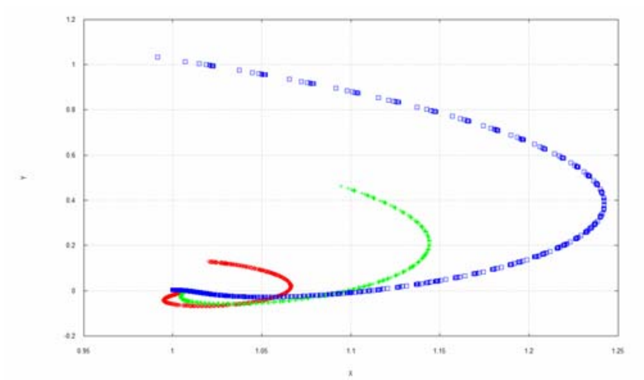

Figure 7 Direct orbits $C_3 = -0.001$, $\alpha = 30^0, 45^0$ and 90^0.

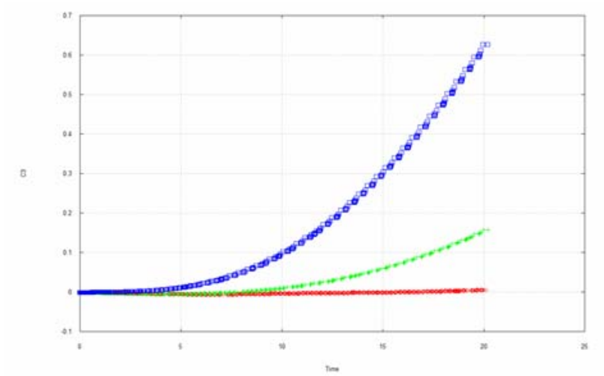

Figure 8 Energy as a function of the time

5 Minimum Value of the Initial Energy

The value of the initial energy is related to the minimum consumption of fuel by the space vehicle. Below it is shown an analysis of the minimum value of the initial energy of the space vehicle in a trajectory that ends in gravitational capture.
In the two graphs below we supplied in the vertical axis the initial value of the energy of the space vehicle C_3 and in the horizontal axis the value of the initial angle α of the vehicle in degrees.
The value of C_3 supplied in these graphs is the smallest value that provides gravitational capture for a given angle α.
The angle α varies from 0^0 to 360^0 in steps of 1^0. The value of the energy varies from -0.1 to -0.001 with steps of 0.001.
In Fig. 9 we have the graphs in red are for retrograde orbits and in green for direct orbits.

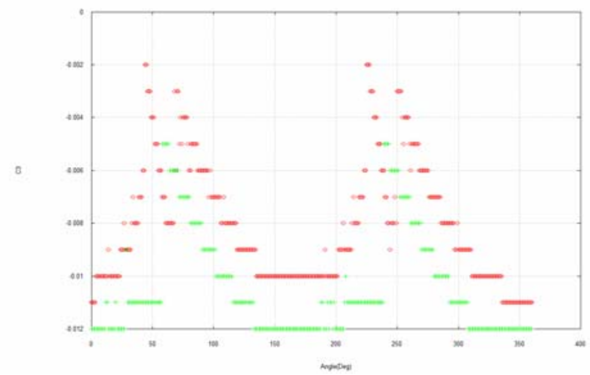

Figure 9 minimum value of the initial energy

6 Conclusions

In this article it was made a computational analysis of the gravitational capture for the system Neptune-Triton and a space vehicle. We used as our model the circular restricted problem of three bodies. We calculated trajectories of gravitational capture for the direct motion as well as for the retrograde motion. It was made the analysis of the minimum value of the initial energy of the space vehicle that allow us to obtain gravitational capture.

References:

[1] Belbruno, E.A., "Examples of the Nonlinear Dynamics of Ballistic Capture and Escape in the Earth-Moon System", *AIAA-90-2896. In: AIAA Astrodynamics Conference*, Portland, Oregon, Aug. 1990.

[2] Yamakawa, H., Kawaguchi, J., Ishii, N., Matsuo, H., "On Earth-Moon transfer trajectory with gravitational capture", *AAS paper 93-633, AAS/AIAA Astrodynamics Specialist Conference*, Victoria, Canada, 1993.

[3] Belbruno, E.A., "Lunar Capture Orbits, a Method of Constructing Earth Moon Trajectories and the Lunar Gas Mission", *AIAA-87-1054. In: 19th AIAA/DGLR/JSASS International Electric Propulsion Conference,* Colorado Springs, Colorado, May 1987.

[4] Belbruno, E.A., Miller, J.K., "A Ballistic Lunar Capture Trajectory for Japanese Spacecraft Hiten", *Jet Propulsion Lab., JPL IOM 312/90.4-1731, Internal Document*, Pasadena, CA, Jun. 1990.

[5] Yamakawa, H., Kawaguchi, J., Ishii, N., Matsuo, H., "A Numerical Study of Gravitational Capture Orbit in Earth-Moon System", *AAS paper 92-186, AAS/AIAA Spaceflight Mechanics Meeting*, Colorado Springs, Colorado, 1992.

[6] Simo, C., Gomez, G., Jorba, A. and Masdemont, J., “The Bicircular model near the triangular libration points of the RTBP”, *A.E. Roy and B.A. Steves, editors, From Newton to Chaos,* pp. 343-370, New York, 1995. Plenum Press.

[7] Vieira Neto, E. and Prado, A.F.B.A., A Study of the Gravitational Capture in the Restricted-Problem. *Proceedings of the "International Symposium on Space Dynamics"* pg. 613-622. Toulouse, France, 19-23/June/1995.

[8] Vieira Neto, E. and Prado, A.F.B.A., Study of the Gravitational Capture in the Elliptical Restricted Three-Body Problem. *Proceedings of the "International Symposium on Space Dynamics"* pg. 202-207. Gifu, Japan, 19-25/May/1996.

[9] Belbruno, E.A., The Dynamical Mechanism of Ballistic Lunar Capture Transfers in the Four-Body Problem from the Perspective of Invariant Manifolds and Hill´s Regions, Institut D´Estudis Catalans, CRM Research Report Nº 270.

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