DYNAMICS DAYS SOUTH AMERICA 2010
International Conference on Chaos and Nonlinear Dynamics

INPE – National Institute for Space Research
São José dos Campos – SP – Brazil – July 26-30, 2010

AMPLITUDE-PHASE SYNCHRONIZATION IN INTERMITTENT TURBULENCE AND SPATIOTEMPORAL CHAOS

Rodrigo A. Miranda[1,2], *Abraham C.-L. Chian*[1], *Daiki Koga*[1], *Erico L. Rempel*[2], *Yoshitaka Saiki*[3], *Michio Yamada*[4]

[1]National Institute for Space Research (INPE), São José dos Campos-SP, Brazil. E-mail: rmiranda@dge.inpe.br
[2]Institute of Aeronautical Technology (IEFM/ITA), São José dos Campos-SP, Brazil
[3]Hokkaido University, Sapporo, Hokkaido 0600812, Japan
[4]Research Institute for Mathematical Sciences (RIMS), Kyoto University, Kyoto, Japan

keywords: Synchronization; Fluid Dynamics, Plasma and Turbulence; Applications of Nonlinear Sciences; Time series analysis; Chaotic Dynamics.

The solar-terrestrial environment provides a natural laboratory for observing intermittent turbulence [1]. The degree of non-Gaussianity (intermittency) in a turbulence can be quantified by calculating the normalized fourth-order structure function, also known as kurtosis. Recently, a phase coherence surrogate technique for characterizing phase synchronization was developed for space plasmas [2]. The link between non-Gaussianity and phase synchronization in intermittent turbulence was established by Koga et al. [3] using magnetic field data upstream and downstream of the Earth's bow shock.

The analysis of dynamical systems modelled by partial differential equations (PDEs) serves as a bridge between chaos theory and both plasma and fluid dynamics. Such systems may exhibit a wealth of regimes, which include temporal chaos (TC) and spatiotemporal chaos (STC) [6]. In PDEs, we refer to temporal chaos whenever the patterns generated vary chaotically in time, but spatial coherence is preserved. In spatiotemporal chaos, the dynamics is chaotic in time and irregular in space. At the onset of STC, there is on-off spatiotemporal intermittency which consists of random switchings between TC and STC [6].

In this paper we analyze synchronization due to multiscale interactions in observations of intermittent turbulence and numerical simulations of spatiotemporal intermittency. First, we apply kurtosis and phase coherence index to measure the degree of amplitude-phase synchronization of intermittent magnetic field turbulence observed in the solar wind. Next, we use a model of nonlinear waves to measure the degree of amplitude-phase synchronization by computing the power-phase spectral entropy, respectively, at the onset of spatiotemporal intermittency. Our results indicate that the duality of amplitude-phase synchronization may be the origin of intermittency in fully-developed turbulence in the solar-terrestrial environment.

1. INTERMITTENT MAGNETIC FIELD TURBULENCE

We analyze the modulus of interplanetary magnetic field $|\mathbf{B}|$ detected by ACE and Cluster from 19:40:40 UT on 1 February 2002 to 03:56:38 UT on 3 February 2002. During this interval Cluster is in the solar wind upstream of the Earth's bow shock [4]. The degree of intermittency and non-Gaussianity of $|\mathbf{B}|$ can be quantified by calculating kurtosis

$$K(\tau) = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{\delta B(\tau) - \langle \delta B(\tau)\rangle}{\sigma_B}\right) - 3, \qquad (1)$$

where $\delta B(\tau) = |\mathbf{B}(t+\tau)| - |\mathbf{B}(t)|$, $\langle\ \rangle$ denote the mean value, and σ_B denote the standard deviation of δB. A Gaussian signal gives $K(\tau) = 0$ for all τ, whereas for an intermittent signal $K(\tau) > 0$ and $K(\tau)$ increases as scale decreases within the inertial subrange. Since K is a quantity computed from the magnetic field fluctuations at scale τ raised to the fourth power, it is proportional to the magnetic energy squared. Hence, kurtosis can be regarded as a quantity which represents the degree of amplitude synchronization.

Phase synchronization among scales can be quantified by the phase coherence index [2]. This index measures the degree of phase synchronization in an original data set (B_{ORG}) by comparing it with a phase-randomized surrogate (B_{PRS}) and a phase-correlated surrogate (B_{PCS}) as follows

$$C_\phi(\tau) = \frac{S_{PRS}(\tau) - S_{ORG}(\tau)}{S_{PRS}(\tau) - S_{PCS}(\tau)} \qquad (2)$$

where $S_j(\tau) = \sum_{i=1}^{N} |B_j(t+\tau) - B_j(t)|$ with $j = ORG$, PRS and PCS. $C_\phi(\tau) = 0$ indicates that the phases of the original data are completely random at scale τ, whereas $C_\phi(\tau) = 1$ indicates that the phases are fully correlated at scale τ.

Figure 1 shows the variation of kurtosis (upper panel) and the phase coherence index (lower panel) as a function of time scale τ for magnetic field fluctuations of ACE and Cluster. For 10 s $\lesssim \tau \lesssim 10^3$s, both kurtosis and the phase coherence

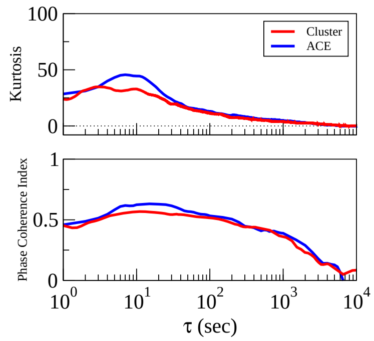

Figure 1 – Kurtosis and phase coherence index of $|\mathbf{B}|$ measured by Cluster and ACE as a function of time scale τ.

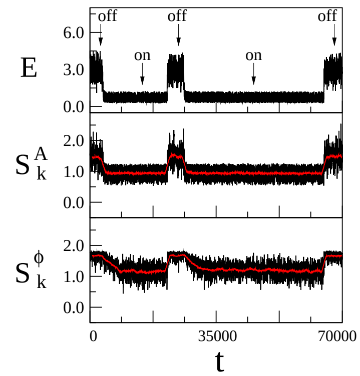

Figure 2 – Time series of E, S_k^A and S_k^ϕ for on-off spatiotemporal intermittency at $\varepsilon = 0.20005$. The red lines denote averaged curves.

index increase as the time scale decreases, which demonstrates the duality of amplitude and phase synchronization measured by $K(\tau)$ and $C_\phi(\tau)$ respectively.

2. INTERMITTENT SPATIOTEMPORAL CHAOS

The Benjamin-Bona-Mahony equation which describes nonlinear long-waves in a dispersive medium such as nonlinear drift waves in magnetized plasmas [5, 6] is given by

$$\partial_t u + c\partial_x u + fu\partial_x u + a\partial_{txx} u = -\nu u - \varepsilon \sin(\kappa x - \Omega t), \quad (3)$$

where ε is the driver amplitude, $c = 1$, $f = -6$, $a = -0.287$, $\nu = 0.1$, $\kappa = 1$ and $\Omega = 0.65$ [6]. Equation (3) is solved numerically using the pseudospectral method by expanding $u(x,t)$ in a Fourier series as $u(x,t) = \sum_{k=-N}^{N} \hat{u}_k(t) \exp(ikx)$, where $\hat{u}_k(t)$ denotes the complex Fourier coefficients.

We quantify the degree of amplitude synchronization related to multiscale interactions by the Fourier power spectral entropy [6]

$$S_k^A(t) = -\sum_{k=1}^{N} p(\hat{u}_k(t)) \ln[p(\hat{u}_k(t))], \quad (4)$$

where $p(\hat{u}_k(t)) = |\hat{u}_k(t)|^2 / \sum_{k=1}^{N} |\hat{u}_k(t)|^2$. The degree of phase synchronization can be quantified by the Fourier phase spectral entropy [7]

$$S_k^\phi(t) = -\sum_{k=1}^{N} P(\delta\phi_k(t)) \ln[P(\delta\phi_k(t))] \quad (5)$$

where P denotes the probability distribution function of phase differences $\delta\phi_k(t) = \phi_{k+1}(t) - \phi_k(t)$, and $\phi_k(t) = \arctan[\mathrm{Im}(\hat{u}_k(t))/\mathrm{Re}(\hat{u}_k(t))]$.

Eq. (3) exhibits a transition from TC to STC at $\varepsilon \sim 0.2$ [6]. At the onset of STC, the time series of wave energy $E(t)$ displays on-off (TC-STC) intermittency shown in the upper panel of Fig. 2 for $\varepsilon = 0.20005$. The middle and lower panels of Fig. 2 show the time series of $S_k^A(t)$ and $S_k^\phi(t)$, respectively. The duality of amplitude and phase synchronization in the on-off spatiotemporal intermittency, confirmed by $S_k^A(t)$ and $S_k^\phi(t)$, is clearly demonstrated in Fig. 2.

3. CONCLUSION

In this paper we measured the degree of amplitude-phase synchronization in intermittent magnetic field turbulence by computing kurtosis and the phase coherence index, and in spatiotemporal intermittency using the Fourier power-phase spectral entropy. The observational and theoretical results presented here demonstrate the duality of amplitude-phase synchronization which is responsible for intermittency in fully-developed turbulence in the solar-terrestrial environment.

4. ACKNOWLEDGEMENTS

This work is supported by CNPq, FAPESP and RIMS.

References

[1] Y. Kamide, and A. C.-L. Chian (Eds.), “Handbook of the Solar-Terrestrial Environment” (Berlin, Springer), 2007.

[2] T. Hada, D. Koga, and E. Yamamoto, Space Sci. Rev. 107, 463, 2003.

[3] D. Koga, A. C.-L. Chian, R. A. Miranda, and E. L. Rempel, Phys. Rev. E 75, 046401, 2007.

[4] A. C.-L. Chian and R. A. Miranda, Ann. Geophys. 27, 1789, 2009.

[5] T. B. Benjamin, J. L. Bona, and J. J. Mahony, Philos. Trans. R. Soc. London Ser. A 272, 47 (1972).

[6] E. L. Rempel, R. A. Miranda, and A. C.-L. Chian. Phys. Fluids 21, 074105 (2009); A. C.-L. Chian, R. A. Miranda, E. L. Rempel, Y. Saiki, and M. Yamada, Phys. Rev. Lett. submitted.

[7] P. Tass et al., Phys. Rev. Lett. 81, 3291 (1998); Y.-C. Lai, M. G. Frei, and I. Osorio, Phys. Rev. E 73, 026214 (2006).