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NEURAL NETWORKS FOR EMULATION VARIATIONAL METHOD FOR DATA ASSIMILATION IN NONLINEAR DYNAMICS

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Description of a physical phenomenon through differential equations presents errors involved, since the mathematical model is always an approximation of reality. For an operational prediction system, one strategy to improve the prediction is to add some information from the real dynamics into mathematical model. This aditional information consists of observations on the phenomenon. However, the observational data insertion should be done carefully, for avoiding a worse performance of the prediction.

Technical data assimilation are tools to combine data from physical-mathematics model with observational data to obtain a better forecast. From filtering point of view, the data assimilation process can be represented by:

$$w^a = w^f + Kp[y - H(w^f)] \tag{1}$$

where w^a is the value of the analysis; w^f is the forecasting (from the mathematical model, also known as background field); K is the weight matrix; y denotes the observation; H represents the observation system; the difference $\{y - H(w^f)\}$ is the innovation; and $p[.]$ is a discrepancy function. Another approach for solving the data assimilation is by computing the minimum solution for the cost function:

$$J(w) = \langle (w - w^f), C(w - w^f) \rangle + \langle [y - H(w^f)], S[y - H(w^f)] \rangle \tag{2}$$

where C and S denote the inverse covariance matrices of the background (in general such matrix is represented by B or Q for the data assimilation or for the control theory communities, respectively) and of the measurement errors (denoted by R), and $\langle u, v \rangle$ expresses the internal product between vectors u and v. For the Kalman filter (KF), the matrix K (Kalman gain) at equation 1 is computed from a formula involving matrices B and R. Variational scheme is linked to the optimization problem with the objective function given by expression by equation 2.

It is very difficult to estimate the background matrix B (also called the modeling error covariance matrix). There are many techniques to estimate this matrix: using some type of parameterization [4], employing ensemble KF, or a Fokker-Planck equation [2, 3]. For applying the KF [1], some linearization is necessary, and the random variables are assumed to have Gaussian distribution.

To consider a more general problem (nonlinear models and non-Gaussian distribution) the particle filter technique has been proposed [5]. However, this approach has a greater computational complexity than extended KF (EKF). The latter technique has a similar complexity to the 4-dimensional variational (4D-Var) scheme.

Recently, we have introduced a new approach based on artificial neural network (ANN) for data assimilation [4, 6, 7]. In this approach, the ANN emulates the KF. For the present paper, it is addressed an emulation of variational method by artificial neural networks [8]. It is important to mention that ANN has a lower computational cost (complexity) than extended and linear KF, variational method, and particle filter, after training. The Lorenz system under chaotic regime will be used to illustrate the method.

The celebrated Lorenz model has been employed as a test standard for examining the performance of data assimilation methods. The nonlinear systems of three differential equations are:

$$\frac{dx}{d\tau} = -\sigma(x - y) \tag{3}$$

$$\frac{dy}{d\tau} = \rho x - y - xz \tag{4}$$

$$\frac{dz}{d\tau} = xy - \beta z \tag{5}$$

where $\tau = \pi^2 H^{-2}(1 + a^2)\kappa t$ is the non-dimensional time, being $H,\ a,\ \kappa$ and t respectively layer height, thermal conductivity, wave number (diameter of the Rayleigh-Bérnard cell), and time; $\sigma,\ \beta$ and ρ are constant.

The results of data assimilation by neural network a compared with variational technique. Figure 1 show the result of data assimilation by artificial neural network. Figure 2 show error; **left side**: error by variational method; **right side**: error by neural network. Artificial Neural Networks (ANN) have become important tools for information processing [9].

Much research has been conducted in pursuing new neural network models and adapting the existing ones to solve real life problems, such as those in engineering [9]. ANN are made of arrangements of processing elements called neurons. The artificial neuron model basically consists of a linear combiner followed by an activation function, given by:

$$y_k = \varphi\left(\sum_{j=1}^{n} w_{kj}x_j + b_k\right) \tag{6}$$

where w_{kj} are the connection weights, b_k is a threshold parameter, x_j is the input vector and y_k is the output of the k_{th} neuron.

The Lorenz systems was integrated using a second order Runge Kutta methods, with $\Delta t = 10^3$, with initial condition for the Lorenz system: $w_0 = [x_0 \;\; y_0 \;\; z_0]^T = [1.508870 \; 1.531271 \; 25.46091]^T$. For training data set, 2000 data are considered. The results are also analyzed with other data assimilation methods. Figure 3 shows the error (absolute difference between the truth and the estimated) for data observations inserted the each time-step 12.

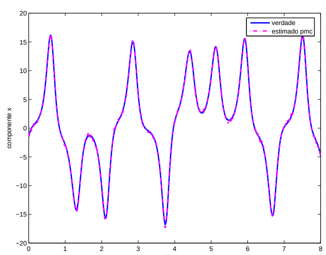

Figure 1 – Temporary series of the component x. (solid curve) truth and (dash curve) estimated by artificial neural network.

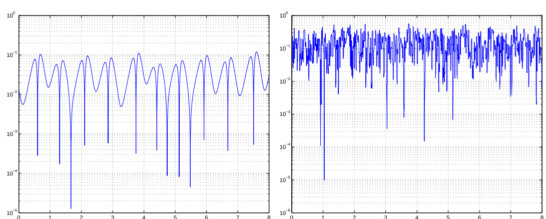

Figure 2 – Graph of the errors at scale logaritmic (absolute difference between the truth and the estimated). left side: error by variational method; right side: error by neural network.

Figure 3 (left hand side) shows the errors for KF, variational technique, and particle filter; right hand side shows errors for ANN emulating this methods.

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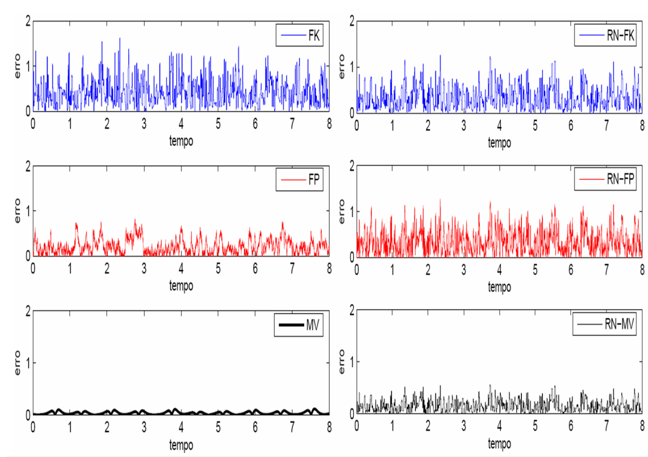

Figure 3 – Graph of the errors, with observations every 12 time steps. left side: errors by KF, particle filter, and varitional method, respectively; right side: errors by ANN.

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